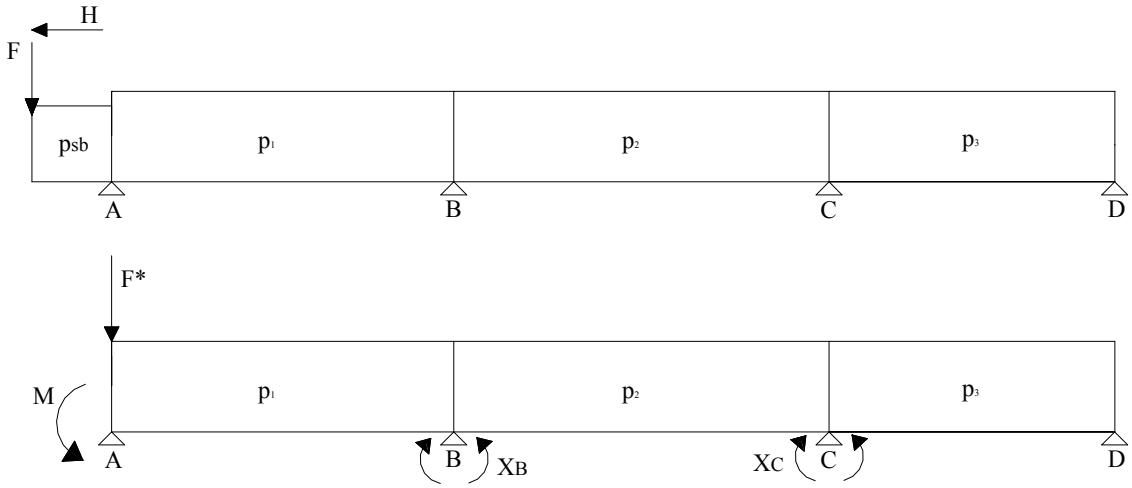
Risoluzione con il metodo delle forze



$F^* = F + p_{sb} \bullet L_{sb}$

$M = F \bullet L_{sb} + p_{sb} \bullet \frac{L_{sb}^2}{2} + H \bullet h$

$$\varphi_{BA} = \frac{M L_1}{6EI} - \frac{p_1 L_1^3}{24EI} + \frac{X_B L_1}{3EI}$$

$$\varphi_{BC} = \frac{p_2 L_2^3}{24EI} - \frac{X_B L_2}{3EI} - \frac{X_C L_2}{6EI}$$

$$\varphi_{CB} = \frac{X_B L_2}{6EI} - \frac{p_2 L_2^3}{24EI} + \frac{X_C L_2}{3EI}$$

$$\varphi_{CD} = \frac{p_3 L_3^3}{24EI} - \frac{X_C L_3}{3EI}$$

$$\varphi_{BA} - \varphi_{BC} = \frac{M L_1}{6EI} - \frac{p_1 L_1^3}{24EI} + \frac{X_B L_1}{3Ei} - \frac{p_2 L_2^3}{24EI} + \frac{X_B L_2}{3EI} + \frac{X_C L_2}{6EI} = 0$$

$$\varphi_{CB} - \varphi_{CD} = \frac{X_B L_2}{6EI} - \frac{p_2 L_2^3}{24EI} + \frac{X_C L_2}{3EI} - \frac{p_3 L_3^3}{24EI} + \frac{X_C L_3}{3EI} = 0$$

$$\frac{X_B L_1}{3Ei} + \frac{X_B L_2}{3EI} + \frac{X_C L_2}{6EI} = \frac{p_1 L_1^3}{24EI} + \frac{p_2 L_2^3}{24EI} - \frac{M L_1}{6EI}$$

$$\frac{X_B L_2}{6EI} + \frac{X_C L_2}{3EI} + \frac{X_C L_3}{3EI} = \frac{p_2 L_2^3}{24EI} + \frac{p_3 L_3^3}{24EI}$$

portando in forma matriciale e semplificando EI :

$$\begin{bmatrix} \frac{L_1}{3}+\frac{L_2}{3} & \frac{L_2}{6} \\ \frac{L_2}{6} & \frac{L_2}{3}+\frac{L_3}{3} \end{bmatrix} \text{x} \begin{bmatrix} X_B \\ X_C \end{bmatrix} = \begin{bmatrix} \frac{p_1 L_1^3}{24}+\frac{p_2 L_2^3}{24}-\frac{M L_1}{6} \\ \frac{p_2 L_2^3}{24}+\frac{p_3 L_3^3}{24} \end{bmatrix}$$

$$\mathbf{D} \text{ x } \mathbf{X^{comb}} = \boldsymbol{\delta}^{\mathbf{comb}} \quad \Rightarrow \quad \mathbf{X^{comb}} = \mathbf{D^{-1}} \text{ x } \boldsymbol{\delta}^{\mathbf{comb}}$$

$$\mathbf{D} = \begin{bmatrix} 3.6000 & 0.9417 \\ 0.9417 & 3.3167 \end{bmatrix}$$

$$\mathbf{D^{-1}} = \begin{bmatrix} 0.3001 & -0.0852 \\ -0.0852 & 0.3257 \end{bmatrix}$$

$$\begin{bmatrix} X_B \\ X_C \end{bmatrix} = \begin{bmatrix} 0.3001 & -0.0852 \\ -0.0852 & 0.3257 \end{bmatrix} \text{x} \begin{bmatrix} \frac{p_1 L_1^3}{24}+\frac{p_2 L_2^3}{24}-\frac{M L_1}{6} \\ \frac{p_2 L_2^3}{24}+\frac{p_3 L_3^3}{24} \end{bmatrix}$$

Luci delle varie campate

L_{SB}= 1.20 m

L_1= 5.15 m

L_2= 5.65 m

L_3= 4.30 m

Carichi

$g_{sk}+g'_{sk}$= 4.06 kN/m $g_k+g'_k$= 5.52 kN/m

q_{sk} = 4.00 kN/m q_k = 3.00 kN/m

F = 0.50 kN

H = 1.00 kN

Combinazione S.L.U. 1

p_{sb}= 4.06 kN/m M = 3.52 kNm

F* = 5.37 kN p_1 = 12.23 kN/m

p_2 = 5.52 kN/m

p_3 = 12.23 kN/m

$$\begin{bmatrix} X_B \\ X_C \end{bmatrix} = \begin{bmatrix} 0.3001 & -0.0852 \\ -0.0852 & 0.3257 \end{bmatrix} * \begin{bmatrix} 108.0664 \\ 81.9987 \end{bmatrix} \quad => \quad \begin{bmatrix} X_B^{(1)} = 25.44 kNm \\ X_C^{(1)} = 17.50 kNm \end{bmatrix}$$

Combinazione S.L.U. 2

p_{sb} = 11.68 kN/m

F* = 14.72 kN

M =10.75 kNm

p_1 = 5.52 kN/m

p_2 =12.23 kN/m

p_3 = 5.52 kN/m

$$\begin{bmatrix} X_B \\ X_C \end{bmatrix} = \begin{bmatrix} 0.3001 & -0.0852 \\ -0.0852 & 0.3257 \end{bmatrix} * \begin{bmatrix} 114.0898 \\ 110.1961 \end{bmatrix} \quad => \quad \begin{bmatrix} X_B^{(2)} = 24.85 kNm \\ X_C^{(2)} = 26.17 kNm \end{bmatrix}$$

Combinazione S.L.U. 3

p_{sb} = 4.06 kN/m

F* = 5.37 kN

M = 3.52 kNm

p_1 = 12.23 kN/m

p_2 = 12.23 kN/m

p_3 = 5.52 kN/m

$$\begin{bmatrix} X_B \\ X_C \end{bmatrix} = \begin{bmatrix} 0.3001 & -0.0852 \\ -0.0852 & 0.3257 \end{bmatrix} * \begin{bmatrix} 158.4926 \\ 110.1961 \end{bmatrix} \quad => \quad \begin{bmatrix} X_B^{(3)} = 38.17 kNm \\ X_C^{(3)} = 22.39 kNm \end{bmatrix}$$

Combinazione S.L.U. 4

p_{sb} = 11.68 kN/m

F* = 14.72 kN

M =10.75 kNm

p_1 = 5.52 kN/m

p_2 = 12.23 kN/m

p_3 = 12.23 kN/m

$$\begin{bmatrix} X_B \\ X_C \end{bmatrix} = \begin{bmatrix} 0.3001 & -0.0852 \\ -0.0852 & 0.3257 \end{bmatrix} * \begin{bmatrix} 114.0984 \\ 133.4250 \end{bmatrix} \quad => \quad \begin{bmatrix} X_B^{(4)} = 22.96 kNm \\ X_C^{(4)} = 33.41 kNm \end{bmatrix}$$

Combinazione S.L.S. 1 (rara)

p_{sb} = 8.06 kN/m M = 7.40 kNm

F* = 10.17 kN p_1 = 8.52 kN/m

p_2 = 8.52 kN/m

p_3 = 8.52 kN/m

$$\begin{bmatrix} X_B \\ X_C \end{bmatrix} = \begin{bmatrix} 0.3001 & -0.0852 \\ -0.0852 & 0.3257 \end{bmatrix} * \begin{bmatrix} 106.1666 \\ 92.2535 \end{bmatrix} \quad => \quad \begin{bmatrix} X_B^{(R)} = 24.00 kNm \\ X_C^{(R)} = 21.00 kNm \end{bmatrix}$$

Combinazione S.L.S. 2 (frequente)

p_{sb} = 6.46 kN/m M = 5.85 kNm

F* = 8.25 kN p_1 = 7.32 kN/m

p_2 = 7.32 kN/m

p_3 = 7.32 kN/m

$$\begin{bmatrix} X_B \\ X_C \end{bmatrix} = \begin{bmatrix} 0.3001 & -0.0852 \\ -0.0852 & 0.3257 \end{bmatrix} * \begin{bmatrix} 91.6494 \\ 79.2601 \end{bmatrix} \quad => \quad \begin{bmatrix} X_B^{(F)} = 20.75 kNm \\ X_C^{(F)} = 18.01 kNm \end{bmatrix}$$

Combinazione S.L.S. 3 (quasi permanente)

p_{sb} = 5.26 kN/m M = 4.69 kNm

F* = 6.81 kN p_1 = 6.42 kN/m

p_2 = 6.42 kN/m

p_3 = 6.42 kN/m

$$\begin{bmatrix} X_B \\ X_C \end{bmatrix} = \begin{bmatrix} 0.3001 & -0.0852 \\ -0.0852 & 0.3257 \end{bmatrix} * \begin{bmatrix} 80.7593 \\ 69.5150 \end{bmatrix} \quad => \quad \begin{bmatrix} X_B^{(Q,P)} = 18.31 kNm \\ X_C^{(Q,P)} = 15.76 kNm \end{bmatrix}$$
