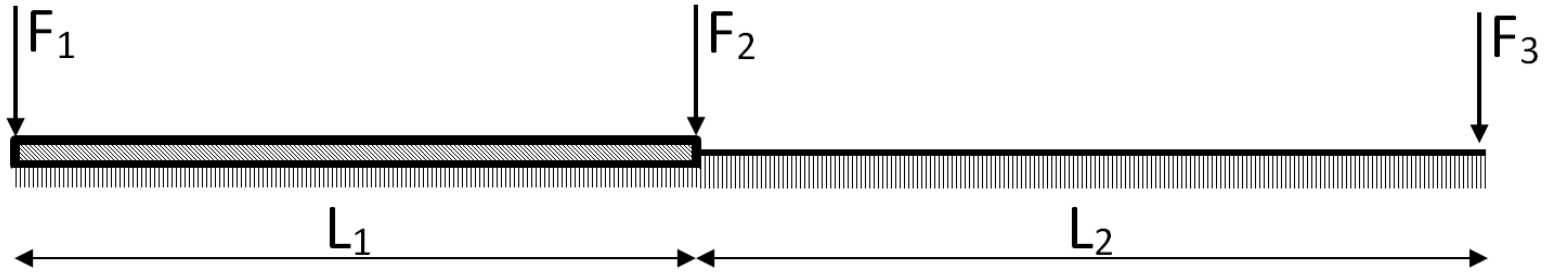
## Schema strutturale




## Soluzione generale

Si introducono le espressioni generali dei campi di spostamenti  relativi ai due tratti del sistema di travi rappresentato sopra.

### ■ Tratto 1

```
In[1]:= Costanti1 = {A0, A1};
        Base1 = {1, x1};
        w1 = Costanti1.Base1
```

```
Out[3]= A0 + A1 x1
```

### ■ Tratto 2

*In[4]:=*
```
Costanti2 = {B1, B2, B3, B4};
Base2 = {Exp[-α x2] Sin[α x2], Exp[-α x2] Cos[α x2], Exp[α x2] Sin[α x2], Exp[α x2] Cos[α x2]};
w2 = Costanti2.Base2
```

*Out[6]=*
$$B2\, e^{-x2\,\alpha}\,\text{Cos}[x2\,\alpha] + B4\, e^{x2\,\alpha}\,\text{Cos}[x2\,\alpha] + B1\, e^{-x2\,\alpha}\,\text{Sin}[x2\,\alpha] + B3\, e^{x2\,\alpha}\,\text{Sin}[x2\,\alpha]$$

---

## Condizioni globali di equilibrio, condizioni al contorno e di accoppiamento

### ■ Equazioni globali di equilibrio sul tratto 1

Agli estremi del tratto infinitamente rigido non è possibile esprimere le caratteristiche della sollecitazione ( Momento Flettente e Taglio) in funzione delle derivate seconda e terza della linea elastica, in quanto non è definito un numero reale che rappresenti la rigidezza flessionale "EI" di quel tratto. Di conseguenza, le derivate seconda e terza sarebbero nulle, mentre le caratteristiche della sollecitazione non lo sono necessariemante: esse assumono valore reale ( non nullo in ragione delle forze nodali effettivamente applicate) e, matematicamente, sono il risultato di un prodotto di una quantità infinitesima (le deritate di ordine superiore di w1) e una infinita (il termine "EI").
Pertanto sul tratto 1 vanno scritte equazioni globali di equilibrio che tengano conto delle azioni e delle reazioni applicate sulla stessa, per effetto della presenza del suolo alla Winkler e del tratto 2.

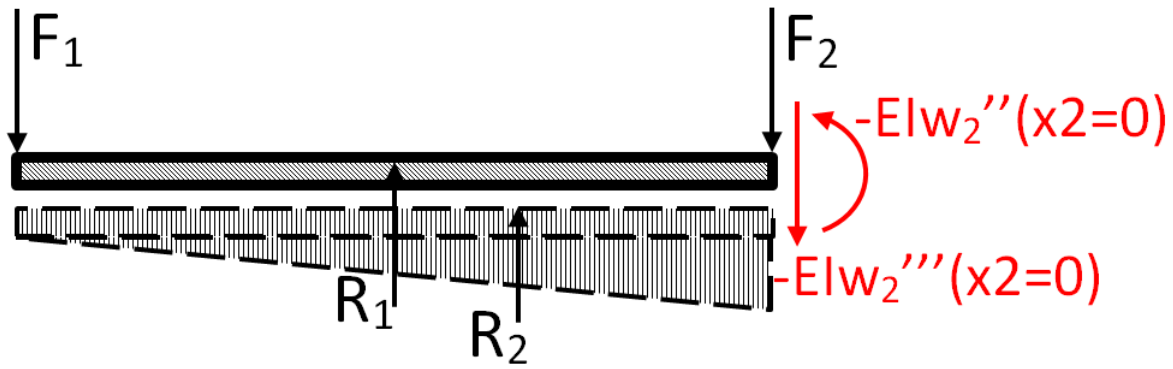



Figura 2: diagramma di corpo libero: azioni e reazioni applicate sul tratto 1.

## ■ Equilibrio globale del tratto 1

```
In[7]:= R1 = k (w1 /. x1 → 0) L1
        R2 = k ((w1 /. x1 → L1) - (w1 /. x1 → 0)) L1 / 2
```

Out[7]= $A0\, k\, L1$

Out[8]= $\frac{1}{2} A1\, k\, L1^2$

```
In[9]:= Eq1 = (-EI D[w2, {x2, 3}] /. x2 → 0) + F1 + F2 - R1 - R2
        Eq2 = (-EI D[w2, {x2, 2}] /. x2 → 0) + F1 L1 - R1 L1 / 2 - R2 L1 / 3
```

Out[9]= $F1 + F2 - A0\, k\, L1 - \frac{1}{2} A1\, k\, L1^2 - EI\, (2\, B1\, \alpha^3 + 2\, B2\, \alpha^3 + 2\, B3\, \alpha^3 - 2\, B4\, \alpha^3)$

Out[10]= $F1\, L1 - \frac{1}{2} A0\, k\, L1^2 - \frac{1}{6} A1\, k\, L1^3 - EI\, (-2\, B1\, \alpha^2 + 2\, B3\, \alpha^2)$

## ■ Congruenza tra i tratti 1 e 2

*In[11]:=*
```
Eq3 = (w2 /. x2 → 0) - (w1 /. x1 → L1)
Eq4 = (-D[w2, x2] /. x2 → 0) - (-D[w1, x1] /. x1 → L1)
```

*Out[11]=*
$$-A0 + B2 + B4 - A1\,L1$$

*Out[12]=*
$$A1 - B1\,\alpha + B2\,\alpha - B3\,\alpha - B4\,\alpha$$

## ■ Condizioni al contorno (statiche) all'estremo del tratto 2 ($x_2$=$L_2$)

*In[13]:=*
```
Eq5 = -EI D[w2, {x2, 2}] /. x2 → L2
Eq6 = (-EI D[w2, {x2, 3}] /. x2 → L2) - F3
```

*Out[13]=*
$$-EI\,\left(-2\,B1\,e^{-L2\,\alpha}\,\alpha^2\,\text{Cos}[L2\,\alpha] + 2\,B3\,e^{L2\,\alpha}\,\alpha^2\,\text{Cos}[L2\,\alpha] + 2\,B2\,e^{-L2\,\alpha}\,\alpha^2\,\text{Sin}[L2\,\alpha] - 2\,B4\,e^{L2\,\alpha}\,\alpha^2\,\text{Sin}[L2\,\alpha]\right)$$

*Out[14]=*
$$-F3 - EI\,\left(2\,B1\,e^{-L2\,\alpha}\,\alpha^3\,\text{Cos}[L2\,\alpha] + 2\,B2\,e^{-L2\,\alpha}\,\alpha^3\,\text{Cos}[L2\,\alpha] + 2\,B3\,e^{L2\,\alpha}\,\alpha^3\,\text{Cos}[L2\,\alpha] - 2\,B4\,e^{L2\,\alpha}\,\alpha^3\,\text{Cos}[L2\,\alpha] + 2\,B1\,e^{-L2\,\alpha}\,\alpha^3\,\text{Sin}[L2\,\alpha] - 2\,B2\,e^{-L2\,\alpha}\,\alpha^3\,\text{Sin}[L2\,\alpha] - 2\,B3\,e^{L2\,\alpha}\,\alpha^3\,\text{Sin}[L2\,\alpha] - 2\,B4\,e^{L2\,\alpha}\,\alpha^3\,\text{Sin}[L2\,\alpha]\right)$$

## ■ Sistema risolutivo

*In[15]:=*
```
Costanti = Join[Costanti1, Costanti2]
BC = {Eq1, Eq2, Eq3, Eq4, Eq5, Eq6}
```

*Out[15]=*
$$\{A0, A1, B1, B2, B3, B4\}$$

*Out[16]=*
$$\left\{F1 + F2 - A0\,k\,L1 - \frac{1}{2}A1\,k\,L1^2 - EI\,\left(2\,B1\,\alpha^3 + 2\,B2\,\alpha^3 + 2\,B3\,\alpha^3 - 2\,B4\,\alpha^3\right),\ F1\,L1 - \frac{1}{2}A0\,k\,L1^2 - \frac{1}{6}A1\,k\,L1^3 - EI\,\left(-2\,B1\,\alpha^2 + 2\,B3\,\alpha^2\right),\ -A0 + B2 + B4 - A1\,L1,\ A1 - B1\,\alpha + B2\,\alpha - B3\,\alpha - B4\,\alpha,\ -EI\,\left(-2\,B1\,e^{-L2\,\alpha}\,\alpha^2\,\text{Cos}[L2\,\alpha] + 2\,B3\,e^{L2\,\alpha}\,\alpha^2\,\text{Cos}[L2\,\alpha] + 2\,B2\,e^{-L2\,\alpha}\,\alpha^2\,\text{Sin}[L2\,\alpha] - 2\,B4\,e^{L2\,\alpha}\,\alpha^2\,\text{Sin}[L2\,\alpha]\right),\ -F3 - EI\,\left(2\,B1\,e^{-L2\,\alpha}\,\alpha^3\,\text{Cos}[L2\,\alpha] + 2\,B2\,e^{-L2\,\alpha}\,\alpha^3\,\text{Cos}[L2\,\alpha] + 2\,B3\,e^{L2\,\alpha}\,\alpha^3\,\text{Cos}[L2\,\alpha] - 2\,B4\,e^{L2\,\alpha}\,\alpha^3\,\text{Cos}[L2\,\alpha] + 2\,B1\,e^{-L2\,\alpha}\,\alpha^3\,\text{Sin}[L2\,\alpha] - 2\,B2\,e^{-L2\,\alpha}\,\alpha^3\,\text{Sin}[L2\,\alpha] - 2\,B3\,e^{L2\,\alpha}\,\alpha^3\,\text{Sin}[L2\,\alpha] - 2\,B4\,e^{L2\,\alpha}\,\alpha^3\,\text{Sin}[L2\,\alpha]\right)\right\}$$

*In[17]:=*
```
CostantiSol = FullSimplify[(Solve[BC == 0, Costanti] // Flatten)]
```

*Out[17]=*

$$\{A0 \to -\frac{2\,(2\,(-2\,F1+F2)\,k\,L1^3\,\alpha+L1\,\alpha\,(-(2\,F1-F2)\,k\,L1^2+12\,EI\,(2\,F1+F2)\,\alpha^2)\,\text{Cos}[2\,L2\,\alpha]+L1\,\alpha\,(-(2\,F1-F2)\,k\,L1^2-12\,EI\,(2\,F1+F2)\,\alpha^2)\,\text{Cosh}[2\,L2\,\alpha]+2\,F3\,\text{Cosh}[L2\,\alpha]\,(2\,k\,L1^3\,\alpha\,\text{Cos}[L2\,\alpha]+3\,(k\,L1^2+4\,EI\,\alpha^2)\,\text{Sin}[L2\,\alpha])+6\,(F3\,((k\,L1^2-4\,EI\,\alpha^2)\,\text{Cos}[L2\,\alpha]+8\,EI\,L1\,\alpha^3\,\text{Sin}[L2\,\alpha])\,\text{Sinh}[L2\,\alpha]+2\,EI\,\alpha^2\,((F1+F2-2\,F1\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]-(F1+F2+2\,F1\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha]))))}{(\alpha\,(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha]+(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha]+8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]+(3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha])))},$$

$$A1 \to \frac{6\,(2\,(-F1+F2)\,k\,L1^2\,\alpha+\alpha\,(-(F1-F2)\,k\,L1^2+4\,EI\,(F1+F2)\,\alpha^2)\,\text{Cos}[2\,L2\,\alpha]-\alpha\,((F1-F2)\,k\,L1^2+4\,EI\,(F1+F2)\,\alpha^2)\,\text{Cosh}[2\,L2\,\alpha]+4\,F3\,k\,L1\,\text{Cosh}[L2\,\alpha]\,(L1\,\alpha\,\text{Cos}[L2\,\alpha]+\text{Sin}[L2\,\alpha])-8\,EI\,F1\,L1\,\alpha^4\,\text{Sin}[2\,L2\,\alpha]+4\,F3\,(k\,L1\,\text{Cos}[L2\,\alpha]+4\,EI\,\alpha^3\,\text{Sin}[L2\,\alpha])\,\text{Sinh}[L2\,\alpha]-8\,EI\,F1\,L1\,\alpha^4\,\text{Sinh}[2\,L2\,\alpha]))}{(\alpha\,(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha]+(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha]+8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]+(3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha])))},$$

$$B1 \to \frac{-e^{L2\,\alpha}\,F3\,k\,L1^2\,(k\,L1^2-12\,EI\,\alpha^2)\,\text{Cos}[L2\,\alpha]+2\,EI\,\alpha^2\,(-3\,(F1-F2)\,k\,L1^2-12\,EI\,(F1+F2)\,\alpha^2+24\,EI\,F1\,L1\,\alpha^3-e^{2\,L2\,\alpha}\,L1\,(3\,(F1-F2)\,k\,L1+(F1-2\,F2)\,k\,L1^2\,\alpha+12\,EI\,F1\,\alpha^3)+\alpha\,((F1-2\,F2)\,k\,L1^3+12\,EI\,(F1+F2)\,\alpha-12\,EI\,F1\,L1\,\alpha^2)\,\text{Cos}[2\,L2\,\alpha])-2\,EI\,L1\,\alpha^2\,(F2\,k\,L1\,(3-2\,L1\,\alpha)+F1\,(12\,EI\,\alpha^3+k\,L1\,(-3+L1\,\alpha)))\,\text{Sin}[2\,L2\,\alpha]+F3\,\text{Sin}[L2\,\alpha]\,(-k\,L1\,(k\,L1^3+12\,EI\,\alpha\,(-2+L1\,\alpha))\,\text{Cosh}[L2\,\alpha]+4\,EI\,\alpha^2\,(12\,EI\,\alpha^2+k\,L1^2\,(3-4\,L1\,\alpha))\,\text{Sinh}[L2\,\alpha]))}{(2\,EI\,\alpha^3\,(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha]+(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha]+8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]+(3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha])))},$$

$$B2 \to \frac{2\,EI\,\alpha^2\,(F2\,(12\,(-1+e^{2\,L2\,\alpha})\,EI\,\alpha^2+k\,L1^2\,(-3+2\,(2+e^{2\,L2\,\alpha})\,L1\,\alpha))+F1\,(-k\,L1^2\,(-3+(2+e^{2\,L2\,\alpha})\,L1\,\alpha)+12\,EI\,\alpha^2\,(-1+e^{2\,L2\,\alpha}\,(1+L1\,\alpha)))-L1\,(F2\,k\,L1\,(3-2\,L1\,\alpha)+F1\,(12\,EI\,\alpha^3+k\,L1\,(-3+L1\,\alpha)))\,\text{Cos}[2\,L2\,\alpha]+6\,e^{L2\,\alpha}\,F3\,(k\,L1^2-4\,EI\,\alpha^2)\,\text{Sin}[L2\,\alpha]+\alpha\,(-(F1-2\,F2)\,k\,L1^3-12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha])-F3\,\text{Cos}[L2\,\alpha]\,(k\,L1^2\,(k\,L1^2+4\,EI\,\alpha^2\,(3-4\,L1\,\alpha))\,\text{Cosh}[L2\,\alpha]-12\,EI\,\alpha\,(4\,EI\,\alpha^3+k\,L1\,(-2+L1\,\alpha))\,\text{Sinh}[L2\,\alpha]))}{(2\,EI\,\alpha^3\,(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha]+(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha]+8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]+(3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha])))},$$

$$B3 \to \frac{(e^{-2\,L2\,\alpha}\,(4\,EI\,\alpha^2\,(-3\,e^{2\,L2\,\alpha}\,((F1-F2)\,k\,L1^2+4\,EI\,(F1+F2)\,\alpha^2+8\,EI\,F1\,L1\,\alpha^3)+L1\,(F2\,k\,L1\,(3-2\,L1\,\alpha)+F1\,(12\,EI\,\alpha^3+k\,L1\,(-3+L1\,\alpha))))+e^{L2\,\alpha}\,(-2\,F3\,k\,L1^2\,(k\,L1^2-12\,EI\,\alpha^2)\,\text{Cos}[L2\,\alpha]+4\,e^{L2\,\alpha}\,EI\,\alpha^3\,(-(F1-2\,F2)\,k\,L1^3+12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2)\,\text{Cos}[2\,L2\,\alpha]+F3\,((1+e^{2\,L2\,\alpha})\,k^2\,L1^4+48\,(-1+e^{2\,L2\,\alpha})\,EI^2\,\alpha^4+8\,EI\,k\,L1\,\alpha\,(3-2\,L1^2\,\alpha^2+e^{2\,L2\,\alpha}\,(3+L1\,\alpha\,(3+2\,L1\,\alpha))))\,\text{Sin}[L2\,\alpha]-4\,e^{L2\,\alpha}\,EI\,L1\,\alpha^2\,(3\,(F1-F2)\,k\,L1+(F1-2\,F2)\,k\,L1^2\,\alpha+12\,EI\,F1\,\alpha^3)\,\text{Sin}[2\,L2\,\alpha])))}{(4\,EI\,\alpha^3\,(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha]+(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha]+8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]+(3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha])))},$$

$$B4 \to \frac{(e^{-2\,L2\,\alpha}\,(4\,EI\,\alpha^2\,(\alpha\,(-(F1-2\,F2)\,k\,L1^3-12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2)-e^{2\,L2\,\alpha}\,L1\,(3\,(F1-F2)\,k\,L1+(F1-2\,F2)\,k\,L1^2\,\alpha+12\,EI\,F1\,\alpha^3)\,\text{Cos}[2\,L2\,\alpha]+6\,e^{L2\,\alpha}\,F3\,(k\,L1^2-4\,EI\,\alpha^2)\,\text{Sin}[L2\,\alpha]+e^{2\,L2\,\alpha}\,(-3\,(F1-F2)\,k\,L1^2-2\,(F1-2\,F2)\,k\,L1^3\,\alpha+12\,EI\,(F1+F2)\,\alpha^2-\alpha\,(-(F1-2\,F2)\,k\,L1^3+12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]))+2\,e^{2\,L2\,\alpha}\,F3\,\text{Cos}[L2\,\alpha]\,(k\,L1^2\,(k\,L1^2+4\,EI\,\alpha^2\,(3+4\,L1\,\alpha))\,\text{Cosh}[L2\,\alpha]+12\,EI\,\alpha\,(4\,EI\,\alpha^3+k\,L1\,(2+L1\,\alpha))\,\text{Sinh}[L2\,\alpha])))}{(4\,EI\,\alpha^3\,(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha]+(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha]+8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]+(3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha])))}\}$$

## Linea elastica

*In[18]:=*
```
w1sol = w1 /. CostantiSol
w2sol = w2 /. CostantiSol
```

*Out[18]=*
$$\begin{aligned}
&(6\,x1\,(2\,(-F1+F2)\,k\,L1^2\,\alpha+\alpha\,(-(F1-F2)\,k\,L1^2+4\,EI\,(F1+F2)\,\alpha^2)\,\text{Cos}[2\,L2\,\alpha]-\\
&\quad\alpha\,((F1-F2)\,k\,L1^2+4\,EI\,(F1+F2)\,\alpha^2)\,\text{Cosh}[2\,L2\,\alpha]+4\,F3\,k\,L1\,\text{Cosh}[L2\,\alpha]\,(L1\,\alpha\,\text{Cos}[L2\,\alpha]+\text{Sin}[L2\,\alpha])-\\
&\quad 8\,EI\,F1\,L1\,\alpha^4\,\text{Sin}[2\,L2\,\alpha]+4\,F3\,(k\,L1\,\text{Cos}[L2\,\alpha]+4\,EI\,\alpha^3\,\text{Sin}[L2\,\alpha])\,\text{Sinh}[L2\,\alpha]-8\,EI\,F1\,L1\,\alpha^4\,\text{Sinh}[2\,L2\,\alpha]))/\\
&(\alpha\,(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha]+(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha]+\\
&\quad 8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]+(3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha])))-\\
&(2\,(2\,(-2\,F1+F2)\,k\,L1^3\,\alpha+L1\,\alpha\,(-(2\,F1-F2)\,k\,L1^2+12\,EI\,(2\,F1+F2)\,\alpha^2)\,\text{Cos}[2\,L2\,\alpha]+L1\,\alpha\,(-(2\,F1-F2)\,k\,L1^2-12\,EI\,(2\,F1+F2)\,\alpha^2)\,\text{Cosh}[2\,L2\,\alpha]+\\
&\quad 2\,F3\,\text{Cosh}[L2\,\alpha]\,(2\,k\,L1^3\,\alpha\,\text{Cos}[L2\,\alpha]+3\,(k\,L1^2+4\,EI\,\alpha^2)\,\text{Sin}[L2\,\alpha])+6\,(F3\,((k\,L1^2-4\,EI\,\alpha^2)\,\text{Cos}[L2\,\alpha]+8\,EI\,L1\,\alpha^3\,\text{Sin}[L2\,\alpha])\,\text{Sinh}[L2\,\alpha]+\\
&\quad\quad 2\,EI\,\alpha^2\,((F1+F2-2\,F1\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]-(F1+F2+2\,F1\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha]))))/\\
&(\alpha\,(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha]+(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha]+\\
&\quad 8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]+(3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha])))
\end{aligned}$$

*Out[19]=*

$$\left(e^{-2\,L2\,\alpha+x2\,\alpha}\left(4\,EI\,\alpha^2\left(-3\,e^{2\,L2\,\alpha}\left((F1-F2)\,k\,L1^2+4\,EI\,(F1+F2)\,\alpha^2+8\,EI\,F1\,L1\,\alpha^3\right)+L1\left(F2\,k\,L1\,(3-2\,L1\,\alpha)+F1\left(12\,EI\,\alpha^3+k\,L1\,(-3+L1\,\alpha)\right)\right)\right)+\right.\right.$$
$$e^{L2\,\alpha}\left(-2\,F3\,k\,L1^2\left(k\,L1^2-12\,EI\,\alpha^2\right)\text{Cos}[L2\,\alpha]+4\,e^{L2\,\alpha}\,EI\,\alpha^3\left(-(F1-2\,F2)\,k\,L1^3+12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2\right)\text{Cos}[2\,L2\,\alpha]+\right.$$
$$F3\left((1+e^{2\,L2\,\alpha})\,k^2\,L1^4+48\left(-1+e^{2\,L2\,\alpha}\right)EI^2\,\alpha^4+8\,EI\,k\,L1\,\alpha\left(3-2\,L1^2\,\alpha^2+e^{2\,L2\,\alpha}\left(3+L1\,\alpha\,(3+2\,L1\,\alpha)\right)\right)\right)\text{Sin}[L2\,\alpha]-$$
$$\left.\left.4\,e^{L2\,\alpha}\,EI\,L1\,\alpha^2\left(3\,(F1-F2)\,k\,L1+(F1-2\,F2)\,k\,L1^2\,\alpha+12\,EI\,F1\,\alpha^3\right)\text{Sin}[2\,L2\,\alpha]\right)\right)\text{Sin}[x2\,\alpha]\Big)\Big/$$
$$\left(4\,EI\,\alpha^3\left(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+\left(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4\right)\text{Cos}[2\,L2\,\alpha]+\left(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4\right)\text{Cosh}[2\,L2\,\alpha]+\right.\right.$$
$$\left.\left.8\,EI\,k\,L1\,\alpha\left(\left(-3+2\,L1^2\,\alpha^2\right)\text{Sin}[2\,L2\,\alpha]+\left(3+2\,L1^2\,\alpha^2\right)\text{Sinh}[2\,L2\,\alpha]\right)\right)\right)+$$
$$\left(e^{-x2\,\alpha}\,\text{Sin}[x2\,\alpha]\left(-e^{L2\,\alpha}\,F3\,k\,L1^2\left(k\,L1^2-12\,EI\,\alpha^2\right)\text{Cos}[L2\,\alpha]+2\,EI\,\alpha^2\left(-3\,(F1-F2)\,k\,L1^2-12\,EI\,(F1+F2)\,\alpha^2+24\,EI\,F1\,L1\,\alpha^3-\right.\right.\right.$$
$$\left.\left.e^{2\,L2\,\alpha}\,L1\left(3\,(F1-F2)\,k\,L1+(F1-2\,F2)\,k\,L1^2\,\alpha+12\,EI\,F1\,\alpha^3\right)+\alpha\left((F1-2\,F2)\,k\,L1^3+12\,EI\,(F1+F2)\,\alpha-12\,EI\,F1\,L1\,\alpha^2\right)\text{Cos}[2\,L2\,\alpha]\right)-$$
$$2\,EI\,L1\,\alpha^2\left(F2\,k\,L1\,(3-2\,L1\,\alpha)+F1\left(12\,EI\,\alpha^3+k\,L1\,(-3+L1\,\alpha)\right)\right)\text{Sin}[2\,L2\,\alpha]+$$
$$F3\,\text{Sin}[L2\,\alpha]\left(-k\,L1\left(k\,L1^3+12\,EI\,\alpha\,(-2+L1\,\alpha)\right)\text{Cosh}[L2\,\alpha]+4\,EI\,\alpha^2\left(12\,EI\,\alpha^2+k\,L1^2\,(3-4\,L1\,\alpha)\right)\text{Sinh}[L2\,\alpha]\right)\Big)\Big/$$
$$\left(2\,EI\,\alpha^3\left(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+\left(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4\right)\text{Cos}[2\,L2\,\alpha]+\left(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4\right)\text{Cosh}[2\,L2\,\alpha]+\right.\right.$$
$$\left.\left.8\,EI\,k\,L1\,\alpha\left(\left(-3+2\,L1^2\,\alpha^2\right)\text{Sin}[2\,L2\,\alpha]+\left(3+2\,L1^2\,\alpha^2\right)\text{Sinh}[2\,L2\,\alpha]\right)\right)\right)+$$
$$\left(e^{-x2\,\alpha}\,\text{Cos}[x2\,\alpha]\left(2\,EI\,\alpha^2\left(F2\left(12\left(-1+e^{2\,L2\,\alpha}\right)EI\,\alpha^2+k\,L1^2\left(-3+2\left(2+e^{2\,L2\,\alpha}\right)L1\,\alpha\right)\right)+\right.\right.\right.$$
$$F1\left(-k\,L1^2\left(-3+\left(2+e^{2\,L2\,\alpha}\right)L1\,\alpha\right)+12\,EI\,\alpha^2\left(-1+e^{2\,L2\,\alpha}\,(1+L1\,\alpha)\right)\right)-L1\left(F2\,k\,L1\,(3-2\,L1\,\alpha)+F1\left(12\,EI\,\alpha^3+k\,L1\,(-3+L1\,\alpha)\right)\right)\text{Cos}[2\,L2\,\alpha]+$$
$$\left.6\,e^{L2\,\alpha}\,F3\left(k\,L1^2-4\,EI\,\alpha^2\right)\text{Sin}[L2\,\alpha]+\alpha\left(-(F1-2\,F2)\,k\,L1^3-12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2\right)\text{Sin}[2\,L2\,\alpha]\right)-$$
$$F3\,\text{Cos}[L2\,\alpha]\left(k\,L1^2\left(k\,L1^2+4\,EI\,\alpha^2\,(3-4\,L1\,\alpha)\right)\text{Cosh}[L2\,\alpha]-12\,EI\,\alpha\left(4\,EI\,\alpha^3+k\,L1\,(-2+L1\,\alpha)\right)\text{Sinh}[L2\,\alpha]\right)\Big)\Big/$$
$$\left(2\,EI\,\alpha^3\left(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+\left(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4\right)\text{Cos}[2\,L2\,\alpha]+\left(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4\right)\text{Cosh}[2\,L2\,\alpha]+\right.\right.$$
$$\left.\left.8\,EI\,k\,L1\,\alpha\left(\left(-3+2\,L1^2\,\alpha^2\right)\text{Sin}[2\,L2\,\alpha]+\left(3+2\,L1^2\,\alpha^2\right)\text{Sinh}[2\,L2\,\alpha]\right)\right)\right)+$$
$$\left(e^{-2\,L2\,\alpha+x2\,\alpha}\,\text{Cos}[x2\,\alpha]\left(4\,EI\,\alpha^2\left(\alpha\left(-(F1-2\,F2)\,k\,L1^3-12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2\right)-\right.\right.\right.$$
$$e^{2\,L2\,\alpha}\,L1\left(3\,(F1-F2)\,k\,L1+(F1-2\,F2)\,k\,L1^2\,\alpha+12\,EI\,F1\,\alpha^3\right)\text{Cos}[2\,L2\,\alpha]+6\,e^{L2\,\alpha}\,F3\left(k\,L1^2-4\,EI\,\alpha^2\right)\text{Sin}[L2\,\alpha]+e^{2\,L2\,\alpha}$$
$$\left.\left.(-3\,(F1-F2)\,k\,L1^2-2\,(F1-2\,F2)\,k\,L1^3\,\alpha+12\,EI\,(F1+F2)\,\alpha^2-\alpha\left(-(F1-2\,F2)\,k\,L1^3+12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2\right)\text{Sin}[2\,L2\,\alpha]\right)\right)+$$
$$2\,e^{2\,L2\,\alpha}\,F3\,\text{Cos}[L2\,\alpha]\left(k\,L1^2\left(k\,L1^2+4\,EI\,\alpha^2\,(3+4\,L1\,\alpha)\right)\text{Cosh}[L2\,\alpha]+12\,EI\,\alpha\left(4\,EI\,\alpha^3+k\,L1\,(2+L1\,\alpha)\right)\text{Sinh}[L2\,\alpha]\right)\Big)\Big/$$
$$\left(4\,EI\,\alpha^3\left(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+\left(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4\right)\text{Cos}[2\,L2\,\alpha]+\left(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4\right)\text{Cosh}[2\,L2\,\alpha]+\right.\right.$$
$$\left.\left.8\,EI\,k\,L1\,\alpha\left(\left(-3+2\,L1^2\,\alpha^2\right)\text{Sin}[2\,L2\,\alpha]+\left(3+2\,L1^2\,\alpha^2\right)\text{Sinh}[2\,L2\,\alpha]\right)\right)\right)$$

## Taglio

Sul tratto 1, il taglio $V_1$ si determina considerando, nella generica sezione di ascissa x1, la risultante delle forze applicate alla sua sinistra (Figura 2). Per il tratto 2, invece, il taglio $V_2$ si ottiene tramite la derivata terza della funzione $w_2$.

*In[20]:=*

```
V1sol = k ((w1sol /. x1 → 0) x1 + (w1sol - (w1sol /. x1 → 0)) x1 / 2) - F1
V2sol = -EI D[w2sol, {x2, 3}]
```

*Out[20]=*

$$
\begin{aligned}
&-F1 + \\
&k\,((3\,x1^2\,(2\,(-F1+F2)\,k\,L1^2\,\alpha+\alpha\,(-(F1-F2)\,k\,L1^2+4\,EI\,(F1+F2)\,\alpha^2)\,\text{Cos}[2\,L2\,\alpha]-\alpha\,((F1-F2)\,k\,L1^2+4\,EI\,(F1+F2)\,\alpha^2)\,\text{Cosh}[2\,L2\,\alpha]+4\,F3\,k\,L1\\
&\quad\text{Cosh}[L2\,\alpha]\,(L1\,\alpha\,\text{Cos}[L2\,\alpha]+\text{Sin}[L2\,\alpha])-8\,EI\,F1\,L1\,\alpha^4\,\text{Sin}[2\,L2\,\alpha]+4\,F3\,(k\,L1\,\text{Cos}[L2\,\alpha]+4\,EI\,\alpha^3\,\text{Sin}[L2\,\alpha])\,\text{Sinh}[L2\,\alpha]-\\
&\quad 8\,EI\,F1\,L1\,\alpha^4\,\text{Sinh}[2\,L2\,\alpha]))/(\alpha\,(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha]+\\
&\quad(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha]+8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]+(3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha])))-\\
&(2\,x1\,(2\,(-2\,F1+F2)\,k\,L1^3\,\alpha+L1\,\alpha\,(-(2\,F1-F2)\,k\,L1^2+12\,EI\,(2\,F1+F2)\,\alpha^2)\,\text{Cos}[2\,L2\,\alpha]+L1\,\alpha\,(-(2\,F1-F2)\,k\,L1^2-12\,EI\,(2\,F1+F2)\,\alpha^2)\,\text{Cosh}[\\
&\quad 2\,L2\,\alpha]+2\,F3\,\text{Cosh}[L2\,\alpha]\,(2\,k\,L1^3\,\alpha\,\text{Cos}[L2\,\alpha]+3\,(k\,L1^2+4\,EI\,\alpha^2)\,\text{Sin}[L2\,\alpha])+6\,(F3\,((k\,L1^2-4\,EI\,\alpha^2)\,\text{Cos}[L2\,\alpha]+8\,EI\,L1\,\alpha^3\,\text{Sin}[L2\,\alpha])\\
&\quad\text{Sinh}[L2\,\alpha]+2\,EI\,\alpha^2\,((F1+F2-2\,F1\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]-(F1+F2+2\,F1\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha])))/\\
&(\alpha\,(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha]+(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha]+\\
&\quad 8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]+(3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha]))))
\end{aligned}
$$

*Out[21]=*

$$
\begin{aligned}
&-EI\,((e^{-2\,L2\,\alpha+x2\,\alpha}\,\text{Cos}[x2\,\alpha]\\
&\quad(4\,EI\,\alpha^2\,(-3\,e^{2\,L2\,\alpha}\,((F1-F2)\,k\,L1^2+4\,EI\,(F1+F2)\,\alpha^2+8\,EI\,F1\,L1\,\alpha^3)+L1\,(F2\,k\,L1\,(3-2\,L1\,\alpha)+F1\,(12\,EI\,\alpha^3+k\,L1\,(-3+L1\,\alpha))))+\\
&\quad e^{L2\,\alpha}\,(-2\,F3\,k\,L1^2\,(k\,L1^2-12\,EI\,\alpha^2)\,\text{Cos}[L2\,\alpha]+4\,e^{L2\,\alpha}\,EI\,\alpha^3\,(-(F1-2\,F2)\,k\,L1^3+12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2)\,\text{Cos}[2\,L2\,\alpha]+\\
&\quad F3\,((1+e^{2\,L2\,\alpha})\,k^2\,L1^4+48\,(-1+e^{2\,L2\,\alpha})\,EI^2\,\alpha^4+8\,EI\,k\,L1\,\alpha\,(3-2\,L1^2\,\alpha^2+e^{2\,L2\,\alpha}\,(3+L1\,\alpha\,(3+2\,L1\,\alpha))))\,\text{Sin}[L2\,\alpha]-\\
&\quad 4\,e^{L2\,\alpha}\,EI\,L1\,\alpha^2\,(3\,(F1-F2)\,k\,L1+(F1-2\,F2)\,k\,L1^2\,\alpha+12\,EI\,F1\,\alpha^3)\,\text{Sin}[2\,L2\,\alpha])))/\\
&(2\,EI\,(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha]+(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha]+\\
&\quad 8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]+(3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha]))-\\
&(e^{-2\,L2\,\alpha+x2\,\alpha}\,(4\,EI\,\alpha^2\,(-3\,e^{2\,L2\,\alpha}\,((F1-F2)\,k\,L1^2+4\,EI\,(F1+F2)\,\alpha^2+8\,EI\,F1\,L1\,\alpha^3)+L1\,(F2\,k\,L1\,(3-2\,L1\,\alpha)+F1\,(12\,EI\,\alpha^3+k\,L1\,(-3+L1\,\alpha))))+\\
&\quad e^{L2\,\alpha}\,(-2\,F3\,k\,L1^2\,(k\,L1^2-12\,EI\,\alpha^2)\,\text{Cos}[L2\,\alpha]+4\,e^{L2\,\alpha}\,EI\,\alpha^3\,(-(F1-2\,F2)\,k\,L1^3+12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2)\,\text{Cos}[2\,L2\,\alpha]+\\
&\quad F3\,((1+e^{2\,L2\,\alpha})\,k^2\,L1^4+48\,(-1+e^{2\,L2\,\alpha})\,EI^2\,\alpha^4+8\,EI\,k\,L1\,\alpha\,(3-2\,L1^2\,\alpha^2+e^{2\,L2\,\alpha}\,(3+L1\,\alpha\,(3+2\,L1\,\alpha))))\,\text{Sin}[L2\,\alpha]-\\
&\quad 4\,e^{L2\,\alpha}\,EI\,L1\,\alpha^2\,(3\,(F1-F2)\,k\,L1+(F1-2\,F2)\,k\,L1^2\,\alpha+12\,EI\,F1\,\alpha^3)\,\text{Sin}[2\,L2\,\alpha]))\,\text{Sin}[x2\,\alpha])/\\
&(2\,EI\,(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha]+(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha]+\\
&\quad 8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]+(3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha]))+\\
&(e^{-x2\,\alpha}\,\text{Cos}[x2\,\alpha]\,(-e^{L2\,\alpha}\,F3\,k\,L1^2\,(k\,L1^2-12\,EI\,\alpha^2)\,\text{Cos}[L2\,\alpha]+2\,EI\,\alpha^2\,(-3\,(F1-F2)\,k\,L1^2-12\,EI\,(F1+F2)\,\alpha^2+24\,EI\,F1\,L1\,\alpha^3-\\
&\quad e^{2\,L2\,\alpha}\,L1\,(3\,(F1-F2)\,k\,L1+(F1-2\,F2)\,k\,L1^2\,\alpha+12\,EI\,F1\,\alpha^3)+\alpha\,((F1-2\,F2)\,k\,L1^3+12\,EI\,(F1+F2)\,\alpha-12\,EI\,F1\,L1\,\alpha^2)\,\text{Cos}[2\,L2\,\alpha])-\\
&\quad 2\,EI\,L1\,\alpha^2\,(F2\,k\,L1\,(3-2\,L1\,\alpha)+F1\,(12\,EI\,\alpha^3+k\,L1\,(-3+L1\,\alpha)))\,\text{Sin}[2\,L2\,\alpha]+\\
&\quad F3\,\text{Sin}[L2\,\alpha]\,(-k\,L1\,(k\,L1^3+12\,EI\,\alpha\,(-2+L1\,\alpha))\,\text{Cosh}[L2\,\alpha]+4\,EI\,\alpha^2\,(12\,EI\,\alpha^2+k\,L1^2\,(3-4\,L1\,\alpha))\,\text{Sinh}[L2\,\alpha])))/\\
&(EI\,(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha]+(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha]+\\
&\quad 8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]+(3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha]))+\\
&(e^{-x2\,\alpha}\,\text{Sin}[x2\,\alpha]\,(-e^{L2\,\alpha}\,F3\,k\,L1^2\,(k\,L1^2-12\,EI\,\alpha^2)\,\text{Cos}[L2\,\alpha]+2\,EI\,\alpha^2\,(-3\,(F1-F2)\,k\,L1^2-12\,EI\,(F1+F2)\,\alpha^2+24\,EI\,F1\,L1\,\alpha^3-\\
&\quad e^{2\,L2\,\alpha}\,L1\,(3\,(F1-F2)\,k\,L1+(F1-2\,F2)\,k\,L1^2\,\alpha+12\,EI\,F1\,\alpha^3)+\alpha\,((F1-2\,F2)\,k\,L1^3+12\,EI\,(F1+F2)\,\alpha-12\,EI\,F1\,L1\,\alpha^2)\,\text{Cos}[2\,L2\,\alpha])-\\
&\quad 2\,EI\,L1\,\alpha^2\,(F2\,k\,L1\,(3-2\,L1\,\alpha)+F1\,(12\,EI\,\alpha^3+k\,L1\,(-3+L1\,\alpha)))\,\text{Sin}[2\,L2\,\alpha]+\\
&\quad F3\,\text{Sin}[L2\,\alpha]\,(-k\,L1\,(k\,L1^3+12\,EI\,\alpha\,(-2+L1\,\alpha))\,\text{Cosh}[L2\,\alpha]+4\,EI\,\alpha^2\,(12\,EI\,\alpha^2+k\,L1^2\,(3-4\,L1\,\alpha))\,\text{Sinh}[L2\,\alpha])))/\\
&(EI\,(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha]+(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha]+
\end{aligned}
$$

$$8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]+(3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha]))) +$$
$$(e^{-x2\,\alpha}\,\text{Cos}[x2\,\alpha]\,(2\,EI\,\alpha^2\,(F2\,(12\,(-1+e^{2\,L2\,\alpha})\,EI\,\alpha^2+k\,L1^2\,(-3+2\,(2+e^{2\,L2\,\alpha})\,L1\,\alpha))+$$
$$F1\,(-k\,L1^2\,(-3+(2+e^{2\,L2\,\alpha})\,L1\,\alpha)+12\,EI\,\alpha^2\,(-1+e^{2\,L2\,\alpha}\,(1+L1\,\alpha)))-L1\,(F2\,k\,L1\,(3-2\,L1\,\alpha)+F1\,(12\,EI\,\alpha^3+k\,L1\,(-3+L1\,\alpha)))$$
$$\text{Cos}[2\,L2\,\alpha]+6\,e^{L2\,\alpha}\,F3\,(k\,L1^2-4\,EI\,\alpha^2)\,\text{Sin}[L2\,\alpha]+\alpha\,(-(F1-2\,F2)\,k\,L1^3-12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha])-$$
$$F3\,\text{Cos}[L2\,\alpha]\,(k\,L1^2\,(k\,L1^2+4\,EI\,\alpha^2\,(3-4\,L1\,\alpha))\,\text{Cosh}[L2\,\alpha]-12\,EI\,\alpha\,(4\,EI\,\alpha^3+k\,L1\,(-2+L1\,\alpha))\,\text{Sinh}[L2\,\alpha])))/$$
$$(EI\,(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha]+(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha]+$$
$$8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]+(3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha])))-$$
$$(e^{-x2\,\alpha}\,\text{Sin}[x2\,\alpha]\,(2\,EI\,\alpha^2\,(F2\,(12\,(-1+e^{2\,L2\,\alpha})\,EI\,\alpha^2+k\,L1^2\,(-3+2\,(2+e^{2\,L2\,\alpha})\,L1\,\alpha))+$$
$$F1\,(-k\,L1^2\,(-3+(2+e^{2\,L2\,\alpha})\,L1\,\alpha)+12\,EI\,\alpha^2\,(-1+e^{2\,L2\,\alpha}\,(1+L1\,\alpha)))-L1\,(F2\,k\,L1\,(3-2\,L1\,\alpha)+F1\,(12\,EI\,\alpha^3+k\,L1\,(-3+L1\,\alpha)))$$
$$\text{Cos}[2\,L2\,\alpha]+6\,e^{L2\,\alpha}\,F3\,(k\,L1^2-4\,EI\,\alpha^2)\,\text{Sin}[L2\,\alpha]+\alpha\,(-(F1-2\,F2)\,k\,L1^3-12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha])-$$
$$F3\,\text{Cos}[L2\,\alpha]\,(k\,L1^2\,(k\,L1^2+4\,EI\,\alpha^2\,(3-4\,L1\,\alpha))\,\text{Cosh}[L2\,\alpha]-12\,EI\,\alpha\,(4\,EI\,\alpha^3+k\,L1\,(-2+L1\,\alpha))\,\text{Sinh}[L2\,\alpha])))/$$
$$(EI\,(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha]+(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha]+$$
$$8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]+(3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha])))-$$
$$(e^{-2\,L2\,\alpha+x2\,\alpha}\,\text{Cos}[x2\,\alpha]\,(4\,EI\,\alpha^2\,(\alpha\,(-(F1-2\,F2)\,k\,L1^3-12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2)-$$
$$e^{2\,L2\,\alpha}\,L1\,(3\,(F1-F2)\,k\,L1+(F1-2\,F2)\,k\,L1^2\,\alpha+12\,EI\,F1\,\alpha^3)\,\text{Cos}[2\,L2\,\alpha]+6\,e^{L2\,\alpha}\,F3\,(k\,L1^2-4\,EI\,\alpha^2)\,\text{Sin}[L2\,\alpha]+e^{2\,L2\,\alpha}$$
$$(-3\,(F1-F2)\,k\,L1^2-2\,(F1-2\,F2)\,k\,L1^3\,\alpha+12\,EI\,(F1+F2)\,\alpha^2-\alpha\,(-(F1-2\,F2)\,k\,L1^3+12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]))+$$
$$2\,e^{2\,L2\,\alpha}\,F3\,\text{Cos}[L2\,\alpha]\,(k\,L1^2\,(k\,L1^2+4\,EI\,\alpha^2\,(3+4\,L1\,\alpha))\,\text{Cosh}[L2\,\alpha]+12\,EI\,\alpha\,(4\,EI\,\alpha^3+k\,L1\,(2+L1\,\alpha))\,\text{Sinh}[L2\,\alpha])))/$$
$$(2\,EI\,(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha]+(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha]+$$
$$8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]+(3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha])))-$$
$$(e^{-2\,L2\,\alpha+x2\,\alpha}\,\text{Sin}[x2\,\alpha]\,(4\,EI\,\alpha^2\,(\alpha\,(-(F1-2\,F2)\,k\,L1^3-12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2)-$$
$$e^{2\,L2\,\alpha}\,L1\,(3\,(F1-F2)\,k\,L1+(F1-2\,F2)\,k\,L1^2\,\alpha+12\,EI\,F1\,\alpha^3)\,\text{Cos}[2\,L2\,\alpha]+6\,e^{L2\,\alpha}\,F3\,(k\,L1^2-4\,EI\,\alpha^2)\,\text{Sin}[L2\,\alpha]+e^{2\,L2\,\alpha}$$
$$(-3\,(F1-F2)\,k\,L1^2-2\,(F1-2\,F2)\,k\,L1^3\,\alpha+12\,EI\,(F1+F2)\,\alpha^2-\alpha\,(-(F1-2\,F2)\,k\,L1^3+12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]))+$$
$$2\,e^{2\,L2\,\alpha}\,F3\,\text{Cos}[L2\,\alpha]\,(k\,L1^2\,(k\,L1^2+4\,EI\,\alpha^2\,(3+4\,L1\,\alpha))\,\text{Cosh}[L2\,\alpha]+12\,EI\,\alpha\,(4\,EI\,\alpha^3+k\,L1\,(2+L1\,\alpha))\,\text{Sinh}[L2\,\alpha])))/$$
$$(2\,EI\,(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha]+(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha]+$$
$$8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha]+(3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha]))))$$

## Momento flettente

Similmente, sul tratto 1, il momento flettente $M_1$ si determina considerando, nella generica sezione di ascissa x1, il momento risultante delle forze applicate alla sua sinistra (Figura 2). Per il tratto 2, invece, il momento flettente $M_2$ si ottiene tramite la derivata seconda della funzione $w_2$.

*In[22]:=*
```
M1sol = k ((w1sol /. x1 → 0) x1^2 / 2 + (w1sol - (w1sol /. x1 → 0)) x1^2 / 6) - F1 x1
M2sol = -EI D[w2sol, {x2, 2}]
```

General::spell1 : Possible spelling error: new symbol name "M1sol" is similar to existing symbol "V1sol". More…

*Out[22]=*
$$
\begin{aligned}
&-F1\,x1 + \\
&k\,((x1^3\,(2\,(-F1+F2)\,k\,L1^2\,\alpha + \alpha\,(-(F1-F2)\,k\,L1^2 + 4\,EI\,(F1+F2)\,\alpha^2)\,\text{Cos}[2\,L2\,\alpha] - \alpha\,((F1-F2)\,k\,L1^2 + 4\,EI\,(F1+F2)\,\alpha^2)\,\text{Cosh}[2\,L2\,\alpha] + 4\,F3\,k\,L1 \\
&\quad \text{Cosh}[L2\,\alpha]\,(L1\,\alpha\,\text{Cos}[L2\,\alpha] + \text{Sin}[L2\,\alpha]) - 8\,EI\,F1\,L1\,\alpha^4\,\text{Sin}[2\,L2\,\alpha] + 4\,F3\,(k\,L1\,\text{Cos}[L2\,\alpha] + 4\,EI\,\alpha^3\,\text{Sin}[L2\,\alpha])\,\text{Sinh}[L2\,\alpha] - \\
&\quad 8\,EI\,F1\,L1\,\alpha^4\,\text{Sinh}[2\,L2\,\alpha]))/(\alpha\,(2\,k^2\,L1^4 - 96\,EI^2\,\alpha^4 + (k^2\,L1^4 - 24\,EI\,k\,L1^2\,\alpha^2 + 48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha] + \\
&\quad (k^2\,L1^4 + 24\,EI\,k\,L1^2\,\alpha^2 + 48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha] + 8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha] + (3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha]))) - \\
&\quad (x1^2\,(2\,(-2\,F1+F2)\,k\,L1^3\,\alpha + L1\,\alpha\,(-(2\,F1-F2)\,k\,L1^2 + 12\,EI\,(2\,F1+F2)\,\alpha^2)\,\text{Cos}[2\,L2\,\alpha] + L1\,\alpha\,(-(2\,F1-F2)\,k\,L1^2 - 12\,EI\,(2\,F1+F2)\,\alpha^2)\,\text{Cosh}[ \\
&\quad\quad 2\,L2\,\alpha] + 2\,F3\,\text{Cosh}[L2\,\alpha]\,(2\,k\,L1^3\,\alpha\,\text{Cos}[L2\,\alpha] + 3\,(k\,L1^2 + 4\,EI\,\alpha^2)\,\text{Sin}[L2\,\alpha]) + 6\,(F3\,((k\,L1^2 - 4\,EI\,\alpha^2)\,\text{Cos}[L2\,\alpha] + 8\,EI\,L1\,\alpha^3\,\text{Sin}[L2\,\alpha]) \\
&\quad\quad \text{Sinh}[L2\,\alpha] + 2\,EI\,\alpha^2\,((F1+F2-2\,F1\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha] - (F1+F2+2\,F1\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha])))) / \\
&\quad (\alpha\,(2\,k^2\,L1^4 - 96\,EI^2\,\alpha^4 + (k^2\,L1^4 - 24\,EI\,k\,L1^2\,\alpha^2 + 48\,EI^2\,\alpha^4)\,\text{Cos}[2\,L2\,\alpha] + (k^2\,L1^4 + 24\,EI\,k\,L1^2\,\alpha^2 + 48\,EI^2\,\alpha^4)\,\text{Cosh}[2\,L2\,\alpha] + \\
&\quad\quad 8\,EI\,k\,L1\,\alpha\,((-3+2\,L1^2\,\alpha^2)\,\text{Sin}[2\,L2\,\alpha] + (3+2\,L1^2\,\alpha^2)\,\text{Sinh}[2\,L2\,\alpha]))))
\end{aligned}
$$

General::spell1 : Possible spelling error: new symbol name "M2sol" is similar to existing symbol "V2sol". More…

*Out[23]=*

$$
\begin{aligned}
&-EI\,\Big(\big(e^{-2\,L2\,\alpha+x2\,\alpha}\,\mathrm{Cos}[x2\,\alpha]\\
&\quad\big(4\,EI\,\alpha^2\,\big(-3\,e^{2\,L2\,\alpha}\,\big((F1-F2)\,k\,L1^2+4\,EI\,(F1+F2)\,\alpha^2+8\,EI\,F1\,L1\,\alpha^3\big)+L1\,\big(F2\,k\,L1\,(3-2\,L1\,\alpha)+F1\,\big(12\,EI\,\alpha^3+k\,L1\,(-3+L1\,\alpha)\big)\big)\big)+\\
&\qquad e^{L2\,\alpha}\,\big(-2\,F3\,k\,L1^2\,\big(k\,L1^2-12\,EI\,\alpha^2\big)\,\mathrm{Cos}[L2\,\alpha]+4\,e^{L2\,\alpha}\,EI\,\alpha^3\,\big(-(F1-2\,F2)\,k\,L1^3+12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2\big)\,\mathrm{Cos}[2\,L2\,\alpha]+\\
&\qquad\quad F3\,\big(\big(1+e^{2\,L2\,\alpha}\big)\,k^2\,L1^4+48\,\big(-1+e^{2\,L2\,\alpha}\big)\,EI^2\,\alpha^4+8\,EI\,k\,L1\,\alpha\,\big(3-2\,L1^2\,\alpha^2+e^{2\,L2\,\alpha}\,\big(3+L1\,\alpha\,(3+2\,L1\,\alpha)\big)\big)\big)\,\mathrm{Sin}[L2\,\alpha]-\\
&\qquad\quad 4\,e^{L2\,\alpha}\,EI\,L1\,\alpha^2\,\big(3\,(F1-F2)\,k\,L1+(F1-2\,F2)\,k\,L1^2\,\alpha+12\,EI\,F1\,\alpha^3\big)\,\mathrm{Sin}[2\,L2\,\alpha]\big)\big)\big)\,/\\
&\quad\big(2\,EI\,\alpha\,\big(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+\big(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4\big)\,\mathrm{Cos}[2\,L2\,\alpha]+\big(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4\big)\,\mathrm{Cosh}[2\,L2\,\alpha]+\\
&\qquad 8\,EI\,k\,L1\,\alpha\,\big(\big(-3+2\,L1^2\,\alpha^2\big)\,\mathrm{Sin}[2\,L2\,\alpha]+\big(3+2\,L1^2\,\alpha^2\big)\,\mathrm{Sinh}[2\,L2\,\alpha]\big)\big)-\\
&\quad\big(e^{-x2\,\alpha}\,\mathrm{Cos}[x2\,\alpha]\,\big(-e^{L2\,\alpha}\,F3\,k\,L1^2\,\big(k\,L1^2-12\,EI\,\alpha^2\big)\,\mathrm{Cos}[L2\,\alpha]+2\,EI\,\alpha^2\,\big(-3\,(F1-F2)\,k\,L1^2-12\,EI\,(F1+F2)\,\alpha^2+24\,EI\,F1\,L1\,\alpha^3-\\
&\qquad e^{2\,L2\,\alpha}\,L1\,\big(3\,(F1-F2)\,k\,L1+(F1-2\,F2)\,k\,L1^2\,\alpha+12\,EI\,F1\,\alpha^3\big)+\alpha\,\big((F1-2\,F2)\,k\,L1^3+12\,EI\,(F1+F2)\,\alpha-12\,EI\,F1\,L1\,\alpha^2\big)\,\mathrm{Cos}[2\,L2\,\alpha]\big)-\\
&\qquad 2\,EI\,L1\,\alpha^2\,\big(F2\,k\,L1\,(3-2\,L1\,\alpha)+F1\,\big(12\,EI\,\alpha^3+k\,L1\,(-3+L1\,\alpha)\big)\big)\,\mathrm{Sin}[2\,L2\,\alpha]+\\
&\qquad F3\,\mathrm{Sin}[L2\,\alpha]\,\big(-k\,L1\,\big(k\,L1^3+12\,EI\,\alpha\,(-2+L1\,\alpha)\big)\,\mathrm{Cosh}[L2\,\alpha]+4\,EI\,\alpha^2\,\big(12\,EI\,\alpha^2+k\,L1^2\,(3-4\,L1\,\alpha)\big)\,\mathrm{Sinh}[L2\,\alpha]\big)\big)\big)\,/\\
&\quad\big(EI\,\alpha\,\big(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+\big(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4\big)\,\mathrm{Cos}[2\,L2\,\alpha]+\big(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4\big)\,\mathrm{Cosh}[2\,L2\,\alpha]+\\
&\qquad 8\,EI\,k\,L1\,\alpha\,\big(\big(-3+2\,L1^2\,\alpha^2\big)\,\mathrm{Sin}[2\,L2\,\alpha]+\big(3+2\,L1^2\,\alpha^2\big)\,\mathrm{Sinh}[2\,L2\,\alpha]\big)\big)+\\
&\quad\big(e^{-x2\,\alpha}\,\mathrm{Sin}[x2\,\alpha]\,\big(2\,EI\,\alpha^2\,\big(F2\,\big(12\,\big(-1+e^{2\,L2\,\alpha}\big)\,EI\,\alpha^2+k\,L1^2\,\big(-3+2\,\big(2+e^{2\,L2\,\alpha}\big)\,L1\,\alpha\big)\big)+\\
&\qquad F1\,\big(-k\,L1^2\,\big(-3+\big(2+e^{2\,L2\,\alpha}\big)\,L1\,\alpha\big)+12\,EI\,\alpha^2\,\big(-1+e^{2\,L2\,\alpha}\,(1+L1\,\alpha)\big)\big)-L1\,\big(F2\,k\,L1\,(3-2\,L1\,\alpha)+F1\,\big(12\,EI\,\alpha^3+k\,L1\,(-3+L1\,\alpha)\big)\big)\\
&\qquad \mathrm{Cos}[2\,L2\,\alpha]+6\,e^{L2\,\alpha}\,F3\,\big(k\,L1^2-4\,EI\,\alpha^2\big)\,\mathrm{Sin}[L2\,\alpha]+\alpha\,\big(-(F1-2\,F2)\,k\,L1^3-12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2\big)\,\mathrm{Sin}[2\,L2\,\alpha]\big)-\\
&\qquad F3\,\mathrm{Cos}[L2\,\alpha]\,\big(k\,L1^2\,\big(k\,L1^2+4\,EI\,\alpha^2\,(3-4\,L1\,\alpha)\big)\,\mathrm{Cosh}[L2\,\alpha]-12\,EI\,\alpha\,\big(4\,EI\,\alpha^3+k\,L1\,(-2+L1\,\alpha)\big)\,\mathrm{Sinh}[L2\,\alpha]\big)\big)\big)\,/\\
&\quad\big(EI\,\alpha\,\big(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+\big(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4\big)\,\mathrm{Cos}[2\,L2\,\alpha]+\big(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4\big)\,\mathrm{Cosh}[2\,L2\,\alpha]+\\
&\qquad 8\,EI\,k\,L1\,\alpha\,\big(\big(-3+2\,L1^2\,\alpha^2\big)\,\mathrm{Sin}[2\,L2\,\alpha]+\big(3+2\,L1^2\,\alpha^2\big)\,\mathrm{Sinh}[2\,L2\,\alpha]\big)\big)-\\
&\quad\big(e^{-2\,L2\,\alpha+x2\,\alpha}\,\mathrm{Sin}[x2\,\alpha]\,\big(4\,EI\,\alpha^2\,\big(\alpha\,\big(-(F1-2\,F2)\,k\,L1^3-12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2\big)-\\
&\qquad e^{2\,L2\,\alpha}\,L1\,\big(3\,(F1-F2)\,k\,L1+(F1-2\,F2)\,k\,L1^2\,\alpha+12\,EI\,F1\,\alpha^3\big)\,\mathrm{Cos}[2\,L2\,\alpha]+6\,e^{L2\,\alpha}\,F3\,\big(k\,L1^2-4\,EI\,\alpha^2\big)\,\mathrm{Sin}[L2\,\alpha]+e^{2\,L2\,\alpha}\\
&\qquad\quad\big(-3\,(F1-F2)\,k\,L1^2-2\,(F1-2\,F2)\,k\,L1^3\,\alpha+12\,EI\,(F1+F2)\,\alpha^2-\alpha\,\big(-(F1-2\,F2)\,k\,L1^3+12\,EI\,(F1+F2)\,\alpha+12\,EI\,F1\,L1\,\alpha^2\big)\,\mathrm{Sin}[2\,L2\,\alpha]\big)\big)+\\
&\qquad 2\,e^{2\,L2\,\alpha}\,F3\,\mathrm{Cos}[L2\,\alpha]\,\big(k\,L1^2\,\big(k\,L1^2+4\,EI\,\alpha^2\,(3+4\,L1\,\alpha)\big)\,\mathrm{Cosh}[L2\,\alpha]+12\,EI\,\alpha\,\big(4\,EI\,\alpha^3+k\,L1\,(2+L1\,\alpha)\big)\,\mathrm{Sinh}[L2\,\alpha]\big)\big)\big)\,/\\
&\quad\big(2\,EI\,\alpha\,\big(2\,k^2\,L1^4-96\,EI^2\,\alpha^4+\big(k^2\,L1^4-24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4\big)\,\mathrm{Cos}[2\,L2\,\alpha]+\big(k^2\,L1^4+24\,EI\,k\,L1^2\,\alpha^2+48\,EI^2\,\alpha^4\big)\,\mathrm{Cosh}[2\,L2\,\alpha]+\\
&\qquad 8\,EI\,k\,L1\,\alpha\,\big(\big(-3+2\,L1^2\,\alpha^2\big)\,\mathrm{Sin}[2\,L2\,\alpha]+\big(3+2\,L1^2\,\alpha^2\big)\,\mathrm{Sinh}[2\,L2\,\alpha]\big)\big)\big)\big)
\end{aligned}
$$

# Esempi

## ■ 1°Caso: $L_2=\lambda/8$

```
In[24]:= L1num = 4000;
         L2num = 6000;
         F1num = 500000;
         F2num = 1000000;
         F3num = 500000;
         k0num = 0.01;
         bnum = 1000;
         hnum = 1504.58;
         Ecnum = 30000;
         EInum = Ecnum bnum hnum^3 / 12;
         knum = k0num bnum;
         αnum = (knum / EInum / 4)^.25
         lambdanum = 2 Pi / αnum
```

```
Out[35]= 0.0001309
```

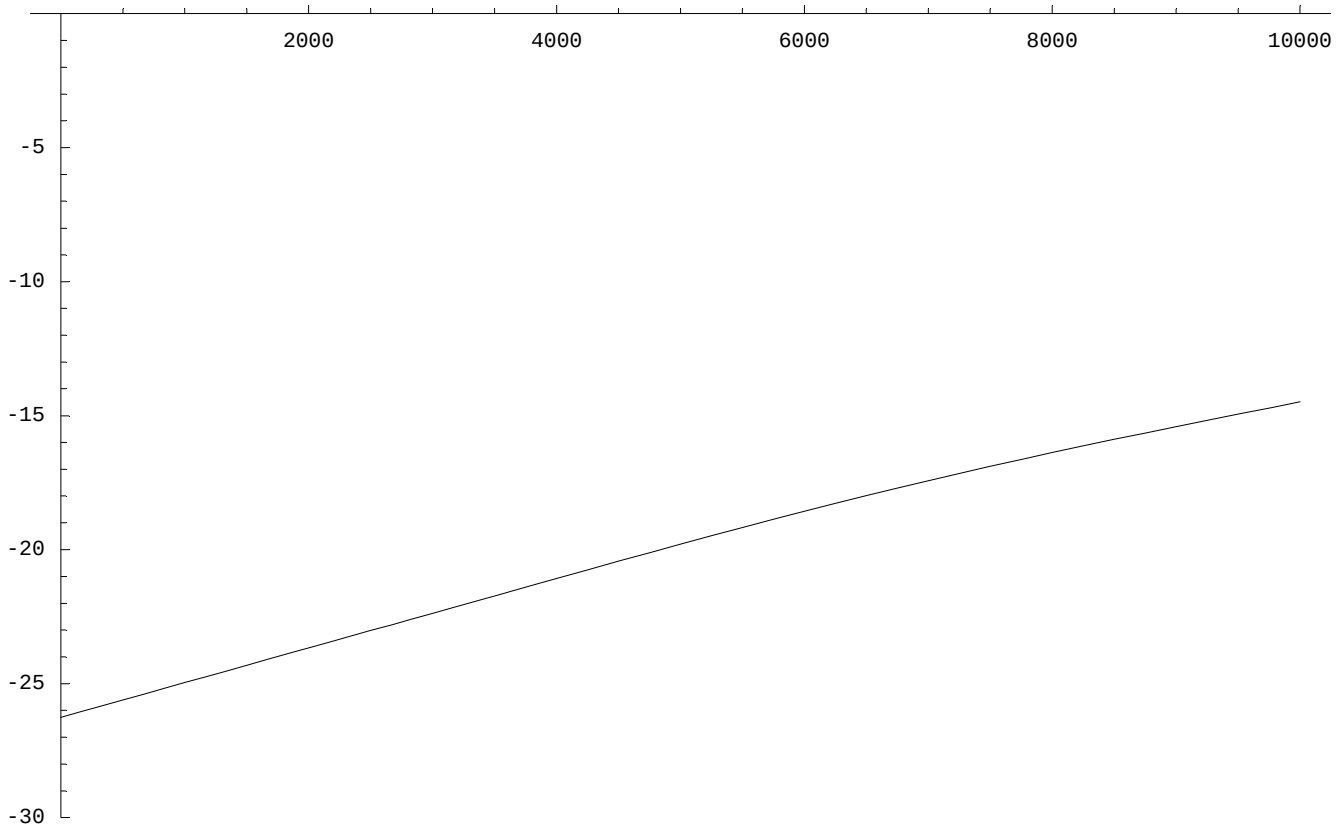
```
Out[36]= 48000.
```

```
In[37]:= w1solNum = w1sol /. {L1 -> L1num, L2 -> L2num, F1 -> F1num, F2 -> F2num, F3 -> F3num, EI -> EInum, k → knum, α → αnum}
         w2solNum = w2sol /. {L1 -> L1num, L2 -> L2num, F1 -> F1num, F2 -> F2num, F3 -> F3num, EI -> EInum, k → knum, α → αnum}
```

```
Out[37]= 26.2563 - 0.00129444 x1
```

```
Out[38]= 21.3858 ℮^(-1.5708+0.0001309 x2) Cos[0.0001309 x2] + 16.6328 ℮^(-0.0001309 x2) Cos[0.0001309 x2] +
          5.83777 ℮^(-1.5708+0.0001309 x2) Sin[0.0001309 x2] + 1.08479 ℮^(-0.0001309 x2) Sin[0.0001309 x2]
```

```
In[39]:= Show[Plot[-w1solNum, {x1, 0, L1num}, PlotRange → {-30, 0}],
           Plot[-w2solNum /. x2 → x - L1num, {x, L1num, L2num + L1num}, PlotRange → {-30, 0}]]
```




```
Out[39]= - Graphics -
```

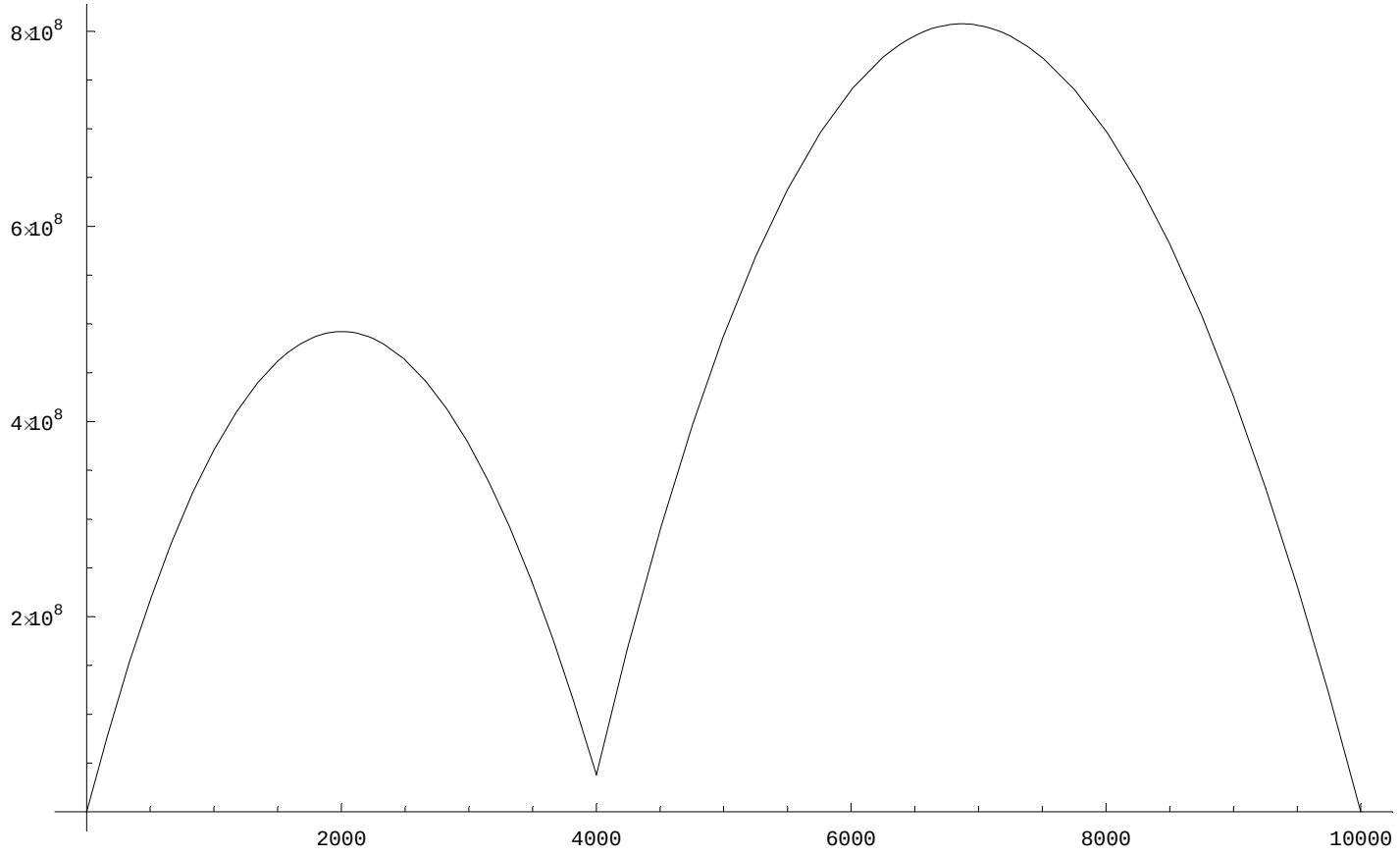
Figura 3: linea elastica del sistema di travi.

*In[40]:=*
```
M1solNum = M1sol /. {L1 -> L1num, L2 -> L2num, F1 -> F1num, F2 -> F2num, F3 -> F3num, EI -> EInum, k → knum, α → αnum}
M2solNum = M2sol /. {L1 -> L1num, L2 -> L2num, F1 -> F1num, F2 -> F2num, F3 -> F3num, EI -> EInum, k → knum, α → αnum}
```

*Out[40]=*
$$-500000\,x1 + 10.\,(13.1281\,x1^2 - 0.00021574\,x1^3)$$

*Out[41]=*
$$-8.51502\times10^{15}\,(2.00057\times10^{-7}\,e^{-1.5708+0.0001309\,x2}\,\text{Cos}[0.0001309\,x2] - 3.71752\times10^{-8}\,e^{-0.0001309\,x2}\,\text{Cos}[0.0001309\,x2] - 7.3288\times10^{-7}\,e^{-1.5708+0.0001309\,x2}\,\text{Sin}[0.0001309\,x2] + 5.69997\times10^{-7}\,e^{-0.0001309\,x2}\,\text{Sin}[0.0001309\,x2])$$

```
In[42]:= Show[Plot[-M1solNum, {x1, 0, L1num}], Plot[-M2solNum /. x2 → x - L1num, {x, L1num, L2num + L1num}]]
```




```
Out[42]= - Graphics -
```

Figura 4:linea elastica del sistema di travi.

Osservazioni: il caso di $L_2=\lambda/8$ determina un comportamento sostanzialmente assimilabile a quello di un'unica trave rigida: si osserva, infatti, che la linea elastica è sostanzialemente rappresentata da un segmento di retta (Figura 3).

## ■ 2°Caso: L= $\lambda$/2

```
In[79]:= L1num = 4000;
         L2num = 24000;
         F1num = 500000;
         F2num = 1000000;
         F3num = 500000;
         k0num = 0.01;
         bnum = 1000;
         hnum = 1504.58;
         Ecnum = 30000;
         EInum = Ecnum bnum hnum^3 / 12;
         knum = k0num bnum;
         αnum = (knum / EInum / 4)^.25
         lambdanum = 2 Pi / αnum
```

```
Out[90]= 0.0001309
```

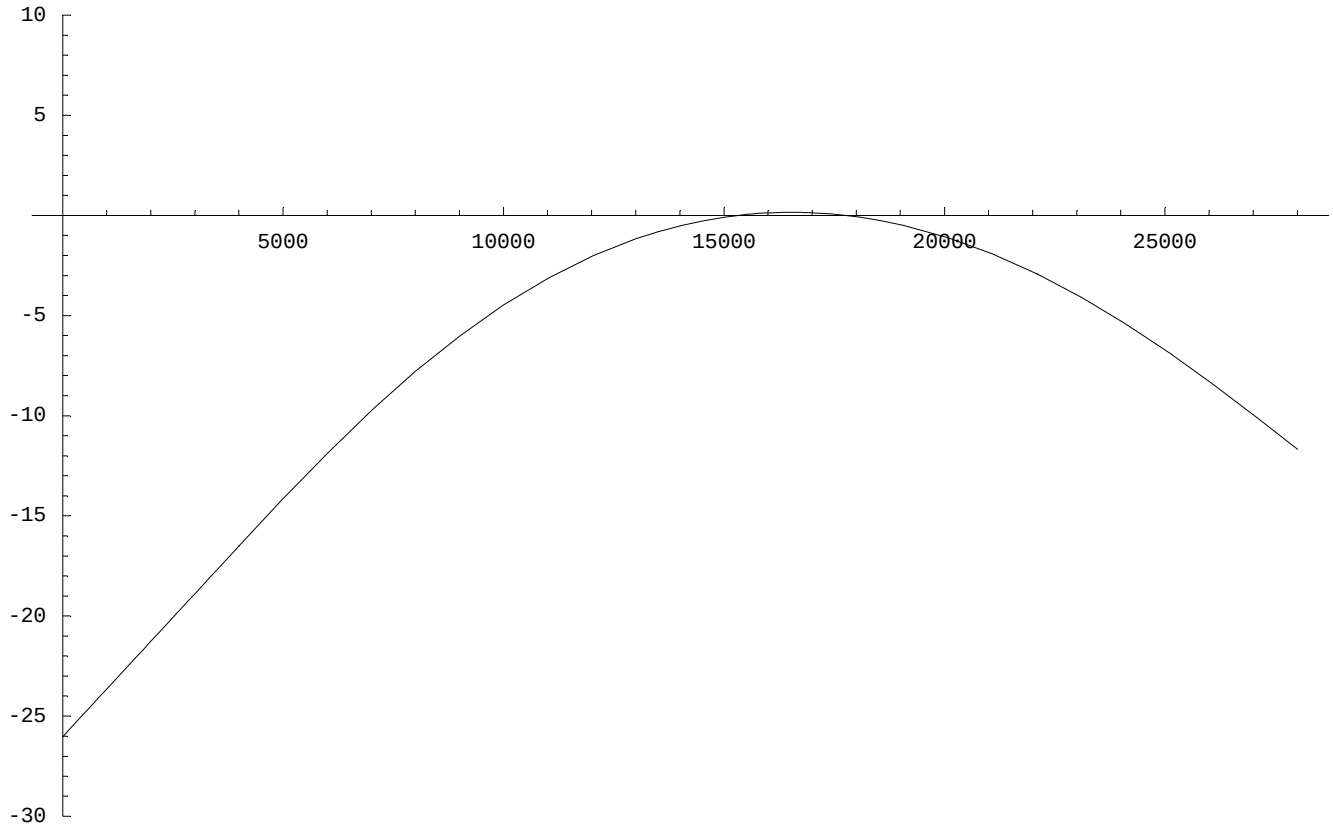
```
Out[91]= 48000.
```

```
In[92]:= w1solNum = w1sol /. {L1 -> L1num, L2 -> L2num, F1 -> F1num, F2 -> F2num, F3 -> F3num, EI -> EInum, k → knum, α → αnum}
         w2solNum = w2sol /. {L1 -> L1num, L2 -> L2num, F1 -> F1num, F2 -> F2num, F3 -> F3num, EI -> EInum, k → knum, α → αnum}
```

```
Out[92]= 26.029 - 0.00237905 x1
```

```
Out[93]= -287.039 ℮^(-6.28318+0.0001309 x2) Cos[0.0001309 x2] + 17.0488 ℮^(-0.0001309 x2) Cos[0.0001309 x2] -
          0.588433 ℮^(-6.28318+0.0001309 x2) Sin[0.0001309 x2] - 0.588641 ℮^(-0.0001309 x2) Sin[0.0001309 x2]
```

```
In[94]:= Show[Plot[-w1solNum, {x1, 0, L1num}, PlotRange → {-30, 10}],
          Plot[-w2solNum /. x2 → x - L1num, {x, L1num, L2num + L1num}, PlotRange → {-30, 10}]]
```




```
Out[94]= - Graphics -
```

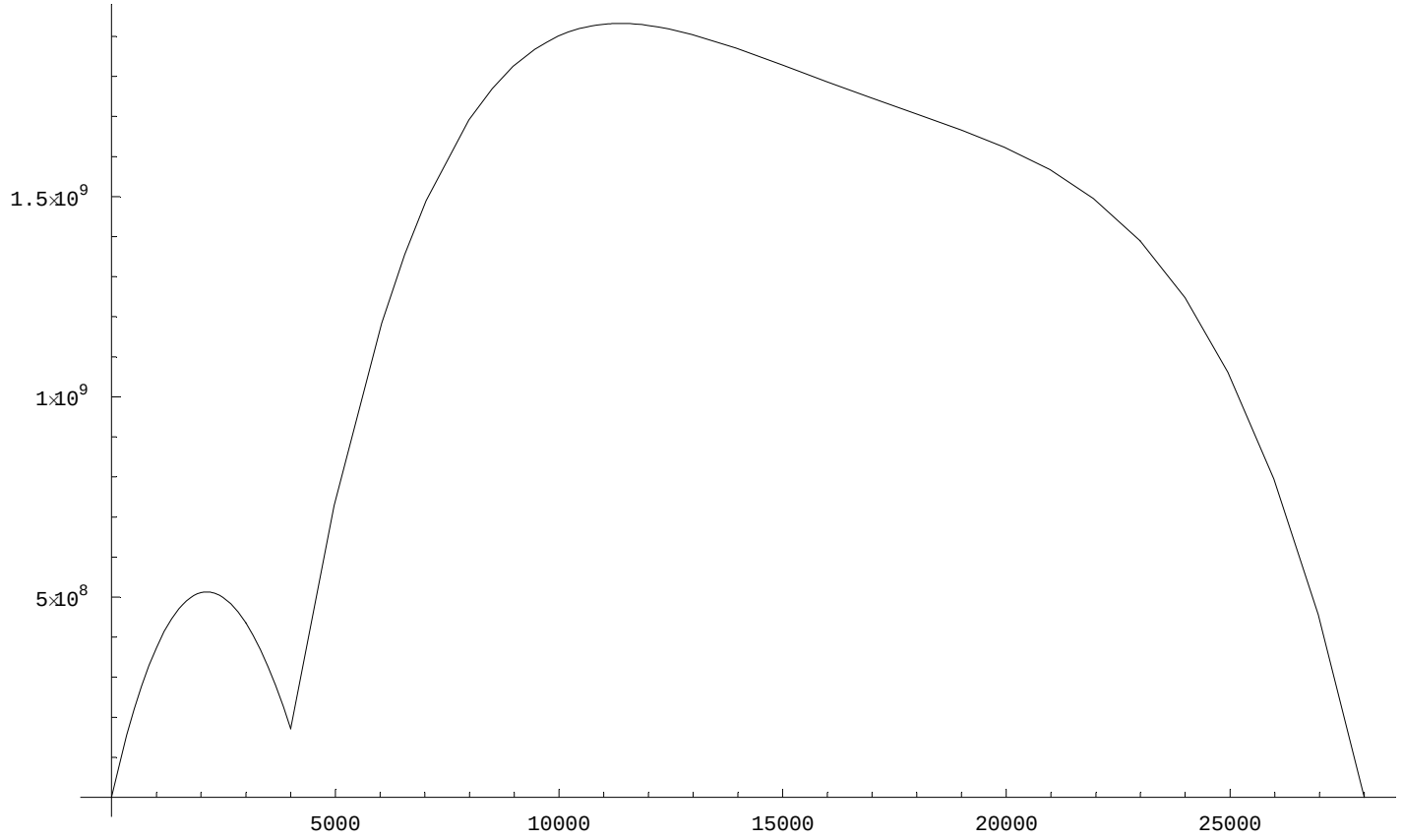
Osservazioni: il caso di $L_2=\lambda/4$ evidenzia la maggiore flessibilità del tratto 2, la cui deformata è evidentemente "non lineare" (Figura 5).

*In[60]:=*
```
M1solNum = M1sol /. {L1 -> L1num, L2 -> L2num, F1 -> F1num, F2 -> F2num, F3 -> F3num, EI -> EInum, k → knum, α → αnum}
M2solNum = M2sol /. {L1 -> L1num, L2 -> L2num, F1 -> F1num, F2 -> F2num, F3 -> F3num, EI -> EInum, k → knum, α → αnum}
```

*Out[60]=*
$$-500000\,x1 + 10.\,(13.0145\,x1^2 - 0.000396508\,x1^3)$$

*Out[61]=*
$$-8.51502\times 10^{15}\,(-2.01653\times 10^{-8}\,e^{-6.28318+0.0001309\,x2}\,\text{Cos}[0.0001309\,x2] + 2.01724\times 10^{-8}\,e^{-0.0001309\,x2}\,\text{Cos}[0.0001309\,x2] + 9.83667\times 10^{-6}\,e^{-6.28318+0.0001309\,x2}\,\text{Sin}[0.0001309\,x2] + 5.84253\times 10^{-7}\,e^{-0.0001309\,x2}\,\text{Sin}[0.0001309\,x2])$$

*In[95]:=*
```
Show[Plot[-M1solNum, {x1, 0, L1num}], Plot[-M2solNum /. x2 → x - L1num, {x, L1num, L2num + L1num}]]
```




*Out[95]=* - Graphics -