Trave di fondazione: confronto tra la soluzione elastica e quella infinitamente rigida.

■ Determinazione della matrice di rigidezza della trave elastica su suolo alla Winkler

```
In[1]:= Funzioni = {Exp[-Alpha z] Sin[Alpha z], Exp[-Alpha z] Cos[Alpha z], Exp[Alpha z] Sin[Alpha z], Exp[Alpha z] Cos[Alpha z]}
        VectA = {A1, A2, A3, A4}
```

Out[1]= $\{e^{-\text{Alpha } z}\,\text{Sin[Alpha z]},\ e^{-\text{Alpha } z}\,\text{Cos[Alpha z]},\ e^{\text{Alpha } z}\,\text{Sin[Alpha z]},\ e^{\text{Alpha } z}\,\text{Cos[Alpha z]}\}$

Out[2]= {A1, A2, A3, A4}

```
In[3]:= w = Funzioni.VectA
```

Out[3]= $\text{A2}\, e^{-\text{Alpha } z}\,\text{Cos[Alpha z]} + \text{A4}\, e^{\text{Alpha } z}\,\text{Cos[Alpha z]} + \text{A1}\, e^{-\text{Alpha } z}\,\text{Sin[Alpha z]} + \text{A3}\, e^{\text{Alpha } z}\,\text{Sin[Alpha z]}$

```
In[4]:= SistemaG = {w /. z → 0, -D[w, z] /. z → 0, w /. z → L, -D[w, z] /. z → L};
        MatG = Table[Table[Simplify[Coefficient[SistemaG[[i]], VectA[[j]]]], {j, 1, 4}], {i, 1, 4}];
        MatrixForm[MatG]
```

Out[6]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 \\ -\text{Alpha} & \text{Alpha} & -\text{Alpha} \\ e^{-\text{Alpha L}}\,\text{Sin[Alpha L]} & e^{-\text{Alpha L}}\,\text{Cos[Alpha L]} & e^{\text{Alpha L}}\,\text{Sin[Alpha L]} \\ \text{Alpha}\, e^{-\text{Alpha L}}\,(-\text{Cos[Alpha L]}+\text{Sin[Alpha L]}) & \text{Alpha}\, e^{-\text{Alpha L}}\,(\text{Cos[Alpha L]}+\text{Sin[Alpha L]}) & -\text{Alpha}\, e^{\text{Alpha L}}\,(\text{Cos[Alpha L]}+\text{Sin[Alpha L]}) & -\text{Alpha} \end{pmatrix}$$

In[7]:=
```
SistemaH = {EI D[w, {z, 3}] /. z → 0, EI D[w, {z, 2}] /. z → 0, -EI D[w, {z, 3}] /. z → L, -EI D[w, {z, 2}] /. z → L};
MatH = Table[Table[Simplify[Coefficient[SistemaH[[i]], VectA[[j]]]], {j, 1, 4}], {i, 1, 4}];
MatrixForm[MatH]
```

Out[9]//MatrixForm=

$$\begin{pmatrix} 2\,\mathrm{Alpha}^3\,\mathrm{EI} & 2\,\mathrm{Alpha}^3\,\mathrm{EI} & 2\,\mathrm{Alpha}^3\,\mathrm{EI} \\ -2\,\mathrm{Alpha}^2\,\mathrm{EI} & 0 & 2\,\mathrm{Alpha}^2\,\mathrm{EI} \\ -2\,\mathrm{Alpha}^3\,e^{-\mathrm{Alpha}\,L}\,\mathrm{EI}\,(\mathrm{Cos}[\mathrm{Alpha}\,L]+\mathrm{Sin}[\mathrm{Alpha}\,L]) & -2\,\mathrm{Alpha}^3\,e^{-\mathrm{Alpha}\,L}\,\mathrm{EI}\,(\mathrm{Cos}[\mathrm{Alpha}\,L]-\mathrm{Sin}[\mathrm{Alpha}\,L]) & -2\,\mathrm{Alpha}^3\,e^{\mathrm{Alpha}\,L}\,\mathrm{EI}\,(\mathrm{Cos}[\mathrm{Alpha}\,L]-\mathrm{Sin}[\\ 2\,\mathrm{Alpha}^2\,e^{-\mathrm{Alpha}\,L}\,\mathrm{EI}\,\mathrm{Cos}[\mathrm{Alpha}\,L] & -2\,\mathrm{Alpha}^2\,e^{-\mathrm{Alpha}\,L}\,\mathrm{EI}\,\mathrm{Sin}[\mathrm{Alpha}\,L] & -2\,\mathrm{Alpha}^2\,e^{\mathrm{Alpha}\,L}\,\mathrm{EI}\,\mathrm{Cos}[\mathrm{Alpha}\,L \end{pmatrix}$$

In[10]:=
```
MatK = Simplify[MatH.Inverse[MatG]];
MatrixForm[MatK]
```

Out[11]//MatrixForm=

$$\begin{pmatrix} \frac{4\,\mathrm{Alpha}^3\,\mathrm{EI}\,(-1+e^{4\,\mathrm{Alpha}\,L}+2\,e^{2\,\mathrm{Alpha}\,L}\,\mathrm{Sin}[2\,\mathrm{Alpha}\,L])}{1-4\,e^{2\,\mathrm{Alpha}\,L}+e^{4\,\mathrm{Alpha}\,L}+2\,e^{2\,\mathrm{Alpha}\,L}\,\mathrm{Cos}[2\,\mathrm{Alpha}\,L]} & -\frac{2\,\mathrm{Alpha}^2\,\mathrm{EI}\,(1+e^{4\,\mathrm{Alpha}\,L}-2\,e^{2\,\mathrm{Alpha}\,L}\,\mathrm{Cos}[2\,\mathrm{Alpha}\,L])}{1-4\,e^{2\,\mathrm{Alpha}\,L}+e^{4\,\mathrm{Alpha}\,L}+2\,e^{2\,\mathrm{Alpha}\,L}\,\mathrm{Cos}[2\,\mathrm{Alpha}\,L]} & -\frac{8\,\mathrm{Alpha}^3\,e^{\mathrm{Alpha}\,L}\,\mathrm{EI}\,((-1+e^{2\,\mathrm{Alpha}\,L})\,\mathrm{Cos}[}{1-4\,e^{2\,\mathrm{Alpha}\,L}+e^{4\,\mathrm{Alpha}\,L}+2\,e} \\ -\frac{2\,\mathrm{Alpha}^2\,\mathrm{EI}\,(1+e^{4\,\mathrm{Alpha}\,L}-2\,e^{2\,\mathrm{Alpha}\,L}\,\mathrm{Cos}[2\,\mathrm{Alpha}\,L])}{1-4\,e^{2\,\mathrm{Alpha}\,L}+e^{4\,\mathrm{Alpha}\,L}+2\,e^{2\,\mathrm{Alpha}\,L}\,\mathrm{Cos}[2\,\mathrm{Alpha}\,L]} & \frac{2\,\mathrm{Alpha}\,\mathrm{EI}\,(-1+e^{4\,\mathrm{Alpha}\,L}-2\,e^{2\,\mathrm{Alpha}\,L}\,\mathrm{Sin}[2\,\mathrm{Alpha}\,L])}{1-4\,e^{2\,\mathrm{Alpha}\,L}+e^{4\,\mathrm{Alpha}\,L}+2\,e^{2\,\mathrm{Alpha}\,L}\,\mathrm{Cos}[2\,\mathrm{Alpha}\,L]} & \frac{8\,\mathrm{Alpha}^2\,e^{\mathrm{Alpha}\,L}\,(-1+e^{2\,\mathrm{Alpha}}}{1-4\,e^{2\,\mathrm{Alpha}\,L}+e^{4\,\mathrm{Alpha}\,L}+2\,e} \\ -\frac{8\,\mathrm{Alpha}^3\,e^{\mathrm{Alpha}\,L}\,\mathrm{EI}\,((-1+e^{2\,\mathrm{Alpha}\,L})\,\mathrm{Cos}[\mathrm{Alpha}\,L]+(1+e^{2\,\mathrm{Alpha}\,L})\,\mathrm{Sin}[\mathrm{Alpha}\,L])}{1-4\,e^{2\,\mathrm{Alpha}\,L}+e^{4\,\mathrm{Alpha}\,L}+2\,e^{2\,\mathrm{Alpha}\,L}\,\mathrm{Cos}[2\,\mathrm{Alpha}\,L]} & \frac{8\,\mathrm{Alpha}^2\,e^{\mathrm{Alpha}\,L}\,(-1+e^{2\,\mathrm{Alpha}\,L})\,\mathrm{EI}\,\mathrm{Sin}[\mathrm{Alpha}\,L]}{1-4\,e^{2\,\mathrm{Alpha}\,L}+e^{4\,\mathrm{Alpha}\,L}+2\,e^{2\,\mathrm{Alpha}\,L}\,\mathrm{Cos}[2\,\mathrm{Alpha}\,L]} & \frac{4\,\mathrm{Alpha}^3\,\mathrm{EI}\,(-1+e^{4\,\mathrm{Alpha}\,L}+2\,e}{1-4\,e^{2\,\mathrm{Alpha}\,L}+e^{4\,\mathrm{Alpha}\,L}+2\,e} \\ -\frac{8\,\mathrm{Alpha}^2\,e^{\mathrm{Alpha}\,L}\,(-1+e^{2\,\mathrm{Alpha}\,L})\,\mathrm{EI}\,\mathrm{Sin}[\mathrm{Alpha}\,L]}{1-4\,e^{2\,\mathrm{Alpha}\,L}+e^{4\,\mathrm{Alpha}\,L}+2\,e^{2\,\mathrm{Alpha}\,L}\,\mathrm{Cos}[2\,\mathrm{Alpha}\,L]} & -\frac{4\,\mathrm{Alpha}\,e^{\mathrm{Alpha}\,L}\,\mathrm{EI}\,((-1+e^{2\,\mathrm{Alpha}\,L})\,\mathrm{Cos}[\mathrm{Alpha}\,L]-(1+e^{2\,\mathrm{Alpha}\,L})\,\mathrm{Sin}[\mathrm{Alpha}\,L])}{1-4\,e^{2\,\mathrm{Alpha}\,L}+e^{4\,\mathrm{Alpha}\,L}+2\,e^{2\,\mathrm{Alpha}\,L}\,\mathrm{Cos}[2\,\mathrm{Alpha}\,L]} & \frac{2\,\mathrm{Alpha}^2\,\mathrm{EI}\,(1+e^{4\,\mathrm{Alpha}\,L}-2\,e}{1-4\,e^{2\,\mathrm{Alpha}\,L}+e^{4\,\mathrm{Alpha}\,L}+2\,e} \end{pmatrix}$$

■ Determinazione del vettore delle azioni di incastro perfetto per la trave elastica su suolo alla Winkler

In[12]:=
```
Vects0 = {q / k, 0, q / k, 0};
VectF0 = -Simplify[MatK.Vects0]
```

Out[13]=

$$\Big\{-\frac{4\,\mathrm{Alpha}^3\,\mathrm{EI}\,q\,(1+e^{2\,\mathrm{Alpha}\,L}-2\,e^{\mathrm{Alpha}\,L}\,\mathrm{Cos}[\mathrm{Alpha}\,L])}{k\,(-1+e^{2\,\mathrm{Alpha}\,L}+2\,e^{\mathrm{Alpha}\,L}\,\mathrm{Sin}[\mathrm{Alpha}\,L])},\ -\frac{2\,\mathrm{Alpha}^2\,\mathrm{EI}\,q\,(1-e^{2\,\mathrm{Alpha}\,L}+2\,e^{\mathrm{Alpha}\,L}\,\mathrm{Sin}[\mathrm{Alpha}\,L])}{k\,(-1+e^{2\,\mathrm{Alpha}\,L}+2\,e^{\mathrm{Alpha}\,L}\,\mathrm{Sin}[\mathrm{Alpha}\,L])},$$
$$-\frac{4\,\mathrm{Alpha}^3\,\mathrm{EI}\,q\,(1+e^{2\,\mathrm{Alpha}\,L}-2\,e^{\mathrm{Alpha}\,L}\,\mathrm{Cos}[\mathrm{Alpha}\,L])}{k\,(-1+e^{2\,\mathrm{Alpha}\,L}+2\,e^{\mathrm{Alpha}\,L}\,\mathrm{Sin}[\mathrm{Alpha}\,L])},\ -\frac{2\,\mathrm{Alpha}^2\,\mathrm{EI}\,q\,(-1+e^{2\,\mathrm{Alpha}\,L}-2\,e^{\mathrm{Alpha}\,L}\,\mathrm{Sin}[\mathrm{Alpha}\,L])}{k\,(-1+e^{2\,\mathrm{Alpha}\,L}+2\,e^{\mathrm{Alpha}\,L}\,\mathrm{Sin}[\mathrm{Alpha}\,L])}\Big\}$$

■ Assemblaggio della matrice di rigidezza globale e del vettore delle azioni nodali equivalenti

■ Inizializzazione

```
In[14]:= Nel = 4;
         NNodi = Nel + 1;
         MatKGlob = Table[Table[0, {j, 1, 2 NNodi}], {i, 1, 2 NNodi}];
         MatrixForm[MatKGlob]
         VectSGlob = Table[0, {j, 1, 2 NNodi}];
         VectFGlob = Table[0, {j, 1, 2 NNodi}];
         VectF0Glob = Table[0, {j, 1, 2 NNodi}];
```

Out[17]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

■ Assemblaggio Primo Tratto

```
In[21]:= Nodo1 = 1;
         Nodo2 = 2;
```

```
In[23]:= L = L0;
```

In[24]:=
```
For[i = 1, i <= 2, For[j = 1, j <= 2, MatKGlob[[2 (Nodo1 - 1) + i, 2 (Nodo1 - 1) + j]] =
    MatKGlob[[2 (Nodo1 - 1) + i, 2 (Nodo1 - 1) + j]] + MatK[[i, j]] /. {l → L, Alpha → Alpha1}; j++]; i++];
For[i = 1, i <= 2, For[j = 1, j <= 2, MatKGlob[[2 (Nodo2 - 1) + i, 2 (Nodo1 - 1) + j]] =
    MatKGlob[[2 (Nodo2 - 1) + i, 2 (Nodo1 - 1) + j]] + MatK[[2 + i, j]] /. {l → L, Alpha → Alpha1}; j++]; i++];
For[i = 1, i <= 2, For[j = 1, j <= 2, MatKGlob[[2 (Nodo1 - 1) + i, 2 (Nodo2 - 1) + j]] =
    MatKGlob[[2 (Nodo1 - 1) + i, 2 (Nodo2 - 1) + j]] + MatK[[i, 2 + j]] /. {l → L, Alpha → Alpha1}; j++]; i++];
For[i = 1, i <= 2, For[j = 1, j <= 2, MatKGlob[[2 (Nodo2 - 1) + i, 2 (Nodo2 - 1) + j]] =
    MatKGlob[[2 (Nodo2 - 1) + i, 2 (Nodo2 - 1) + j]] + MatK[[2 + i, 2 + j]] /. {l → L, Alpha → Alpha1}; j++]; i++];
For[j = 1, j <= 2, VectF0Glob[[2 (Nodo1 - 1) + j]] = VectF0Glob[[2 (Nodo1 - 1) + j]] + VectF0[[j]] /. {l → L, Alpha → Alpha1}; j++];
For[j = 1, j <= 2, VectF0Glob[[2 (Nodo2 - 1) + j]] = VectF0Glob[[2 (Nodo2 - 1) + j]] + VectF0[[2 + j]] /. {l → L, Alpha → Alpha1}; j++];
MatrixForm[MatKGlob]
MatrixForm[VectF0Glob]
```

Out[30]//MatrixForm=

$$
\left(\begin{array}{ccc}
\frac{4\,\text{Alpha1}^3\,\text{EI}\,(-1+e^{4\,\text{Alpha1 L0}}+2\,e^{2\,\text{Alpha1 L0}}\,\text{Sin}[2\,\text{Alpha1 L0}])}{1-4\,e^{2\,\text{Alpha1 L0}}+e^{4\,\text{Alpha1 L0}}+2\,e^{2\,\text{Alpha1 L0}}\,\text{Cos}[2\,\text{Alpha1 L0}]} & -\frac{2\,\text{Alpha1}^2\,\text{EI}\,(1+e^{4\,\text{Alpha1 L0}}-2\,e^{2\,\text{Alpha1 L0}}\,\text{Cos}[2\,\text{Alpha1 L0}])}{1-4\,e^{2\,\text{Alpha1 L0}}+e^{4\,\text{Alpha1 L0}}+2\,e^{2\,\text{Alpha1 L0}}\,\text{Cos}[2\,\text{Alpha1 L0}]} & -\frac{8\,\text{Alpha1}^3\,e^{\text{Alpha1}\cdots}}{1\cdots} \\
-\frac{2\,\text{Alpha1}^2\,\text{EI}\,(1+e^{4\,\text{Alpha1 L0}}-2\,e^{2\,\text{Alpha1 L0}}\,\text{Cos}[2\,\text{Alpha1 L0}])}{1-4\,e^{2\,\text{Alpha1 L0}}+e^{4\,\text{Alpha1 L0}}+2\,e^{2\,\text{Alpha1 L0}}\,\text{Cos}[2\,\text{Alpha1 L0}]} & \frac{2\,\text{Alpha1}\,\text{EI}\,(-1+e^{4\,\text{Alpha1 L0}}-2\,e^{2\,\text{Alpha1 L0}}\,\text{Sin}[2\,\text{Alpha1 L0}])}{1-4\,e^{2\,\text{Alpha1 L0}}+e^{4\,\text{Alpha1 L0}}+2\,e^{2\,\text{Alpha1 L0}}\,\text{Cos}[2\,\text{Alpha1 L0}]} & \frac{8\cdots}{1-\cdots} \\
-\frac{8\,\text{Alpha1}^3\,e^{\text{Alpha1 L0}}\,\text{EI}\,((-1+e^{2\,\text{Alpha1 L0}})\,\text{Cos}[\text{Alpha1 L0}]+(1+e^{2\,\text{Alpha1 L0}})\,\text{Sin}[\text{Alpha1 L0}])}{1-4\,e^{2\,\text{Alpha1 L0}}+e^{4\,\text{Alpha1 L0}}+2\,e^{2\,\text{Alpha1 L0}}\,\text{Cos}[2\,\text{Alpha1 L0}]} & \frac{8\,\text{Alpha1}^2\,e^{\text{Alpha1 L0}}\,(-1+e^{2\,\text{Alpha1 L0}})\,\text{EI}\,\text{Sin}[\text{Alpha1 L0}]}{1-4\,e^{2\,\text{Alpha1 L0}}+e^{4\,\text{Alpha1 L0}}+2\,e^{2\,\text{Alpha1 L0}}\,\text{Cos}[2\,\text{Alpha1 L0}]} & \frac{4\,\text{Alpha1}\cdots}{1-\cdots} \\
-\frac{8\,\text{Alpha1}^2\,e^{\text{Alpha1 L0}}\,(-1+e^{2\,\text{Alpha1 L0}})\,\text{EI}\,\text{Sin}[\text{Alpha1 L0}]}{1-4\,e^{2\,\text{Alpha1 L0}}+e^{4\,\text{Alpha1 L0}}+2\,e^{2\,\text{Alpha1 L0}}\,\text{Cos}[2\,\text{Alpha1 L0}]} & -\frac{4\,\text{Alpha1}\,e^{\text{Alpha1 L0}}\,\text{EI}\,((-1+e^{2\,\text{Alpha1 L0}})\,\text{Cos}[\text{Alpha1 L0}]-(1+e^{2\,\text{Alpha1 L0}})\,\text{Sin}[\text{Alpha1 L0}])}{1-4\,e^{2\,\text{Alpha1 L0}}+e^{4\,\text{Alpha1 L0}}+2\,e^{2\,\text{Alpha1 L0}}\,\text{Cos}[2\,\text{Alpha1 L0}]} & \frac{2\,\text{Alpha1}\cdots}{1-\cdots} \\
0 & 0 & \\
0 & 0 & \\
0 & 0 & \\
0 & 0 & \\
0 & 0 & \\
0 & 0 &
\end{array}\right.
$$

Out[31]//MatrixForm=

$$\begin{pmatrix} -\frac{4\,\text{Alpha1}^3\,\text{EI}\,q\,(1+e^{2\,\text{Alpha1}\,\text{L0}}-2\,e^{\text{Alpha1}\,\text{L0}}\,\text{Cos}[\text{Alpha1}\,\text{L0}])}{k\,(-1+e^{2\,\text{Alpha1}\,\text{L0}}+2\,e^{\text{Alpha1}\,\text{L0}}\,\text{Sin}[\text{Alpha1}\,\text{L0}])} \\ -\frac{2\,\text{Alpha1}^2\,\text{EI}\,q\,(1-e^{2\,\text{Alpha1}\,\text{L0}}+2\,e^{\text{Alpha1}\,\text{L0}}\,\text{Sin}[\text{Alpha1}\,\text{L0}])}{k\,(-1+e^{2\,\text{Alpha1}\,\text{L0}}+2\,e^{\text{Alpha1}\,\text{L0}}\,\text{Sin}[\text{Alpha1}\,\text{L0}])} \\ -\frac{4\,\text{Alpha1}^3\,\text{EI}\,q\,(1+e^{2\,\text{Alpha1}\,\text{L0}}-2\,e^{\text{Alpha1}\,\text{L0}}\,\text{Cos}[\text{Alpha1}\,\text{L0}])}{k\,(-1+e^{2\,\text{Alpha1}\,\text{L0}}+2\,e^{\text{Alpha1}\,\text{L0}}\,\text{Sin}[\text{Alpha1}\,\text{L0}])} \\ -\frac{2\,\text{Alpha1}^2\,\text{EI}\,q\,(-1+e^{2\,\text{Alpha1}\,\text{L0}}-2\,e^{\text{Alpha1}\,\text{L0}}\,\text{Sin}[\text{Alpha1}\,\text{L0}])}{k\,(-1+e^{2\,\text{Alpha1}\,\text{L0}}+2\,e^{\text{Alpha1}\,\text{L0}}\,\text{Sin}[\text{Alpha1}\,\text{L0}])} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

■ Primo Tratto: espressioni numeriche

In[32]:=
```
k = 1.4 10^4;
EI = 1.5419 10^6;
q = 16.25;
```

In[35]:=
```
Alpha1 = (k / (4 EI)) ^.25
MatK01 = MatK /. {L → 1, Alpha → Alpha01};
MatrixForm[MatK01]
VectF001 = VectF0 /. {L → 1, Alpha → Alpha01};
MatrixForm[VectF001]
```

Out[35]= 0.218274

Out[37]//MatrixForm=

$$\begin{pmatrix} \frac{6.1676\times10^{6}\,\text{Alpha01}^{3}\,(-1+e^{4\,\text{Alpha01}}+2\,e^{2\,\text{Alpha01}}\,\text{Sin}[2\,\text{Alpha01}])}{1-4\,e^{2\,\text{Alpha01}}+e^{4\,\text{Alpha01}}+2\,e^{2\,\text{Alpha01}}\,\text{Cos}[2\,\text{Alpha01}]} & -\frac{3.0838\times10^{6}\,\text{Alpha01}^{2}\,(1+e^{4\,\text{Alpha01}}-2\,e^{2\,\text{Alpha01}}\,\text{Cos}[2\,\text{Alpha01}])}{1-4\,e^{2\,\text{Alpha01}}+e^{4\,\text{Alpha01}}+2\,e^{2\,\text{Alpha01}}\,\text{Cos}[2\,\text{Alpha01}]} & -\frac{1.23352\times10^{7}}{} \\ -\frac{3.0838\times10^{6}\,\text{Alpha01}^{2}\,(1+e^{4\,\text{Alpha01}}-2\,e^{2\,\text{Alpha01}}\,\text{Cos}[2\,\text{Alpha01}])}{1-4\,e^{2\,\text{Alpha01}}+e^{4\,\text{Alpha01}}+2\,e^{2\,\text{Alpha01}}\,\text{Cos}[2\,\text{Alpha01}]} & \frac{3.0838\times10^{6}\,\text{Alpha01}\,(-1+e^{4\,\text{Alpha01}}-2\,e^{2\,\text{Alpha01}}\,\text{Sin}[2\,\text{Alpha01}])}{1-4\,e^{2\,\text{Alpha01}}+e^{4\,\text{Alpha01}}+2\,e^{2\,\text{Alpha01}}\,\text{Cos}[2\,\text{Alpha01}]} & \\ -\frac{1.23352\times10^{7}\,\text{Alpha01}^{3}\,e^{\text{Alpha01}}\,((-1+e^{2\,\text{Alpha01}})\,\text{Cos}[\text{Alpha01}]+(1+e^{2\,\text{Alpha01}})\,\text{Sin}[\text{Alpha01}])}{1-4\,e^{2\,\text{Alpha01}}+e^{4\,\text{Alpha01}}+2\,e^{2\,\text{Alpha01}}\,\text{Cos}[2\,\text{Alpha01}]} & \frac{1.23352\times10^{7}\,\text{Alpha01}^{2}\,e^{\text{Alpha01}}\,(-1+e^{2\,\text{Alpha01}})\,\text{Sin}[\text{Alpha01}]}{1-4\,e^{2\,\text{Alpha01}}+e^{4\,\text{Alpha01}}+2\,e^{2\,\text{Alpha01}}\,\text{Cos}[2\,\text{Alpha01}]} & \frac{6.1676}{} \\ -\frac{1.23352\times10^{7}\,\text{Alpha01}^{2}\,e^{\text{Alpha01}}\,(-1+e^{2\,\text{Alpha01}})\,\text{Sin}[\text{Alpha01}]}{1-4\,e^{2\,\text{Alpha01}}+e^{4\,\text{Alpha01}}+2\,e^{2\,\text{Alpha01}}\,\text{Cos}[2\,\text{Alpha01}]} & -\frac{6.1676\times10^{6}\,\text{Alpha01}\,e^{\text{Alpha01}}\,((-1+e^{2\,\text{Alpha01}})\,\text{Cos}[\text{Alpha01}]-(1+e^{2\,\text{Alpha01}})\,\text{Sin}[\text{Alpha01}])}{1-4\,e^{2\,\text{Alpha01}}+e^{4\,\text{Alpha01}}+2\,e^{2\,\text{Alpha01}}\,\text{Cos}[2\,\text{Alpha01}]} & - \end{pmatrix}$$

Out[39]//MatrixForm=

$$\begin{pmatrix} -\frac{7158.82\,\text{Alpha01}^{3}\,(1+e^{2\,\text{Alpha01}}-2\,e^{\text{Alpha01}}\,\text{Cos}[\text{Alpha01}])}{-1+e^{2\,\text{Alpha01}}+2\,e^{\text{Alpha01}}\,\text{Sin}[\text{Alpha01}]} \\ -\frac{3579.41\,\text{Alpha01}^{2}\,(1-e^{2\,\text{Alpha01}}+2\,e^{\text{Alpha01}}\,\text{Sin}[\text{Alpha01}])}{-1+e^{2\,\text{Alpha01}}+2\,e^{\text{Alpha01}}\,\text{Sin}[\text{Alpha01}]} \\ -\frac{7158.82\,\text{Alpha01}^{3}\,(1+e^{2\,\text{Alpha01}}-2\,e^{\text{Alpha01}}\,\text{Cos}[\text{Alpha01}])}{-1+e^{2\,\text{Alpha01}}+2\,e^{\text{Alpha01}}\,\text{Sin}[\text{Alpha01}]} \\ -\frac{3579.41\,\text{Alpha01}^{2}\,(-1+e^{2\,\text{Alpha01}}-2\,e^{\text{Alpha01}}\,\text{Sin}[\text{Alpha01}])}{-1+e^{2\,\text{Alpha01}}+2\,e^{\text{Alpha01}}\,\text{Sin}[\text{Alpha01}]} \end{pmatrix}$$

In[40]:=
```
MatrixForm[MatKGlob] /. {L0 → 1, Alpha0 → Alpha01}
MatrixForm[VectF0Glob] /. {L0 → 1, Alpha0 → Alpha01}
```

Out[40]//MatrixForm=

$$\begin{pmatrix} 1.8508\times10^{7} & -9.25213\times10^{6} & -1.8501\times10^{7} & -9.25097\times10^{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ -9.25213\times10^{6} & 6.16773\times10^{6} & 9.25097\times10^{6} & 3.0837\times10^{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1.8501\times10^{7} & 9.25097\times10^{6} & 1.8508\times10^{7} & 9.25213\times10^{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ -9.25097\times10^{6} & 3.0837\times10^{6} & 9.25213\times10^{6} & 6.16773\times10^{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Out[41]//MatrixForm=

$$\begin{pmatrix} -8.1249 \\ 1.35414 \\ -8.1249 \\ -1.35414 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

■ Assemblaggio Secondo Tratto

In[42]:=
```
Nodo1 = 2;
Nodo2 = 3;
```

In[44]:=
```
L = L1;
```

```
In[45]:= For[i = 1, i <= 2, For[j = 1, j <= 2, MatKGlob[[2 (Nodo1 - 1) + i, 2 (Nodo1 - 1) + j]] =
             MatKGlob[[2 (Nodo1 - 1) + i, 2 (Nodo1 - 1) + j]] + MatK[[i, j]] /. {l → L, Alpha → Alpha1}; j++]; i++];
         For[i = 1, i <= 2, For[j = 1, j <= 2, MatKGlob[[2 (Nodo2 - 1) + i, 2 (Nodo1 - 1) + j]] =
             MatKGlob[[2 (Nodo2 - 1) + i, 2 (Nodo1 - 1) + j]] + MatK[[2 + i, j]] /. {l → L, Alpha → Alpha1}; j++]; i++];
         For[i = 1, i <= 2, For[j = 1, j <= 2, MatKGlob[[2 (Nodo1 - 1) + i, 2 (Nodo2 - 1) + j]] =
             MatKGlob[[2 (Nodo1 - 1) + i, 2 (Nodo2 - 1) + j]] + MatK[[i, 2 + j]] /. {l → L, Alpha → Alpha1}; j++]; i++];
         For[i = 1, i <= 2, For[j = 1, j <= 2, MatKGlob[[2 (Nodo2 - 1) + i, 2 (Nodo2 - 1) + j]] =
             MatKGlob[[2 (Nodo2 - 1) + i, 2 (Nodo2 - 1) + j]] + MatK[[2 + i, 2 + j]] /. {l → L, Alpha → Alpha1}; j++]; i++];
         For[j = 1, j <= 2, VectF0Glob[[2 (Nodo1 - 1) + j]] = VectF0Glob[[2 (Nodo1 - 1) + j]] + VectF0[[j]] /. {l → L, Alpha → Alpha1}; j++];
         For[j = 1, j <= 2, VectF0Glob[[2 (Nodo2 - 1) + j]] = VectF0Glob[[2 (Nodo2 - 1) + j]] + VectF0[[2 + j]] /. {l → L, Alpha → Alpha1}; j++];
         MatrixForm[MatKGlob]
         MatrixForm[VectF0Glob]
```

Out[51]//MatrixForm=

$$
\left(\begin{array}{ccc}
\frac{64139.4\,(-1+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Sin}[0.436549\,L0])}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & -\frac{146924.\,(1+e^{0.873098\,L0}-2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0])}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & \\
-\frac{146924.\,(1+e^{0.873098\,L0}-2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0])}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & \frac{673115.\,(-1+e^{0.873098\,L0}-2\,e^{0.436549\,L0}\,\mathrm{Sin}[0.436549\,L0])}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & \\
-\frac{128279.\,e^{0.218274\,L0}\,((-1+e^{0.436549\,L0})\,\mathrm{Cos}[0.218274\,L0]+(1+e^{0.436549\,L0})\,\mathrm{Sin}[0.218274\,L0])}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & \frac{587695.\,e^{0.218274\,L0}\,(-1+e^{0.436549\,L0})\,\mathrm{Sin}[0.218274\,L0]}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & \frac{64139.4}{1-4\,e^{0.436549}} \\
-\frac{587695.\,e^{0.218274\,L0}\,(-1+e^{0.436549\,L0})\,\mathrm{Sin}[0.218274\,L0]}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & -\frac{1.34623\times10^{6}\,e^{0.218274\,L0}\,((-1+e^{0.436549\,L0})\,\mathrm{Cos}[0.218274\,L0]-(1+e^{0.436549\,L0})\,\mathrm{Sin}[0.218274\,L0])}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & \frac{146924.}{1-4\,e^{0.436549}} \\
0 & 0 & \\
0 & 0 & \\
0 & 0 & \\
0 & 0 & \\
0 & 0 & \\
0 & 0 &
\end{array}\right.
$$

Out[52]//MatrixForm=

$$\begin{pmatrix} -\frac{74.4475\,(1+e^{0.436549\,L0}-2\,e^{0.218274\,L0}\,\mathrm{Cos}[0.218274\,L0])}{-1+e^{0.436549\,L0}+2\,e^{0.218274\,L0}\,\mathrm{Sin}[0.218274\,L0]} \\ -\frac{170.537\,(1-e^{0.436549\,L0}+2\,e^{0.218274\,L0}\,\mathrm{Sin}[0.218274\,L0])}{-1+e^{0.436549\,L0}+2\,e^{0.218274\,L0}\,\mathrm{Sin}[0.218274\,L0]} \\ -\frac{74.4475\,(1+e^{0.436549\,L0}-2\,e^{0.218274\,L0}\,\mathrm{Cos}[0.218274\,L0])}{-1+e^{0.436549\,L0}+2\,e^{0.218274\,L0}\,\mathrm{Sin}[0.218274\,L0]}-\frac{74.4475\,(1+e^{0.436549\,L1}-2\,e^{0.218274\,L1}\,\mathrm{Cos}[0.218274\,L1])}{-1+e^{0.436549\,L1}+2\,e^{0.218274\,L1}\,\mathrm{Sin}[0.218274\,L1]} \\ -\frac{170.537\,(-1+e^{0.436549\,L0}-2\,e^{0.218274\,L0}\,\mathrm{Sin}[0.218274\,L0])}{-1+e^{0.436549\,L0}+2\,e^{0.218274\,L0}\,\mathrm{Sin}[0.218274\,L0]}-\frac{170.537\,(1-e^{0.436549\,L1}+2\,e^{0.218274\,L1}\,\mathrm{Sin}[0.218274\,L1])}{-1+e^{0.436549\,L1}+2\,e^{0.218274\,L1}\,\mathrm{Sin}[0.218274\,L1]} \\ -\frac{74.4475\,(1+e^{0.436549\,L1}-2\,e^{0.218274\,L1}\,\mathrm{Cos}[0.218274\,L1])}{-1+e^{0.436549\,L1}+2\,e^{0.218274\,L1}\,\mathrm{Sin}[0.218274\,L1]} \\ -\frac{170.537\,(-1+e^{0.436549\,L1}-2\,e^{0.218274\,L1}\,\mathrm{Sin}[0.218274\,L1])}{-1+e^{0.436549\,L1}+2\,e^{0.218274\,L1}\,\mathrm{Sin}[0.218274\,L1]} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

■ Secondo Tratto: espressioni numeriche

In[53]:=
```
Alpha01 = (k / (4 EI)) ^.25
MatK02 = MatK /. {L → 4.6, Alpha → Alpha01};
MatrixForm[MatK02]
VectF002 = VectF0 /. {L → 4.6, Alpha → Alpha01};
MatrixForm[VectF002]
```

Out[53]= 0.218274

Out[55]//MatrixForm=

$$\begin{pmatrix} 213917. & -452638. & -181898. & -428129. \\ -452638. & 1.35367\times10^{6} & 428129. & 660744. \\ -181898. & 428129. & 213917. & 452638. \\ -428129. & 660744. & 452638. & 1.35367\times10^{6} \end{pmatrix}$$

Out[57]//MatrixForm=

$$\begin{pmatrix} -37.1657 \\ 28.4478 \\ -37.1657 \\ -28.4478 \end{pmatrix}$$

```
In[58]:= MatrixForm[MatKGlob] /. {L0 → 1, Alpha0 → Alpha01, L1 → 4.6, Alpha1 → Alpha01}
         MatrixForm[VectF0Glob] /. {L0 → 1, Alpha0 → Alpha01, L1 → 4.6, Alpha1 → Alpha01}
```

Out[58]//MatrixForm=

$$\begin{pmatrix} 1.8508\times10^{7} & -9.25213\times10^{6} & -1.8501\times10^{7} & -9.25097\times10^{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ -9.25213\times10^{6} & 6.16773\times10^{6} & 9.25097\times10^{6} & 3.0837\times10^{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1.8501\times10^{7} & 9.25097\times10^{6} & 1.87219\times10^{7} & 8.7995\times10^{6} & -181898. & -428129. & 0 & 0 & 0 & 0 \\ -9.25097\times10^{6} & 3.0837\times10^{6} & 8.7995\times10^{6} & 7.5214\times10^{6} & 428129. & 660744. & 0 & 0 & 0 & 0 \\ 0 & 0 & -181898. & 428129. & 213917. & 452638. & 0 & 0 & 0 & 0 \\ 0 & 0 & -428129. & 660744. & 452638. & 1.35367\times10^{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Out[59]//MatrixForm=

$$\begin{pmatrix} -8.1249 \\ 1.35414 \\ -45.2906 \\ 27.0937 \\ -37.1657 \\ -28.4478 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

■ Assemblaggio Terzo Tratto

```
In[60]:= Nodo1 = 3;
         Nodo2 = 4;
```

```
In[62]:= L = L2;
```

```
In[63]:= For[i = 1, i <= 2, For[j = 1, j <= 2,
            MatKGlob[[2 (Nodo1 - 1) + i, 2 (Nodo1 - 1) + j]] = MatKGlob[[2 (Nodo1 - 1) + i, 2 (Nodo1 - 1) + j]] + MatK[[i, j]] /. l → L; j++]; i++];
         For[i = 1, i <= 2, For[j = 1, j <= 2, MatKGlob[[2 (Nodo2 - 1) + i, 2 (Nodo1 - 1) + j]] =
             MatKGlob[[2 (Nodo2 - 1) + i, 2 (Nodo1 - 1) + j]] + MatK[[2 + i, j]] /. l → L; j++]; i++];
         For[i = 1, i <= 2, For[j = 1, j <= 2, MatKGlob[[2 (Nodo1 - 1) + i, 2 (Nodo2 - 1) + j]] =
             MatKGlob[[2 (Nodo1 - 1) + i, 2 (Nodo2 - 1) + j]] + MatK[[i, 2 + j]] /. l → L; j++]; i++];
         For[i = 1, i <= 2, For[j = 1, j <= 2, MatKGlob[[2 (Nodo2 - 1) + i, 2 (Nodo2 - 1) + j]] =
             MatKGlob[[2 (Nodo2 - 1) + i, 2 (Nodo2 - 1) + j]] + MatK[[2 + i, 2 + j]] /. l → L; j++]; i++];
         For[j = 1, j <= 2, VectF0Glob[[2 (Nodo1 - 1) + j]] = VectF0Glob[[2 (Nodo1 - 1) + j]] + VectF0[[j]] /. l → L; j++];
         For[j = 1, j <= 2, VectF0Glob[[2 (Nodo2 - 1) + j]] = VectF0Glob[[2 (Nodo2 - 1) + j]] + VectF0[[2 + j]] /. l → L; j++];
         MatrixForm[MatKGlob]
         MatrixForm[VectF0Glob]
```

Out[69]//MatrixForm=

$$\left(\begin{array}{ccc}
\frac{64139.4\,(-1+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Sin}[0.436549\,L0])}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & -\frac{146924.\,(1+e^{0.873098\,L0}-2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0])}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & \\
-\frac{146924.\,(1+e^{0.873098\,L0}-2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0])}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & \frac{673115.\,(-1+e^{0.873098\,L0}-2\,e^{0.436549\,L0}\,\mathrm{Sin}[0.436549\,L0])}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & \\
-\frac{128279.\,e^{0.218274\,L0}\,((-1+e^{0.436549\,L0})\,\mathrm{Cos}[0.218274\,L0]+(1+e^{0.436549\,L0})\,\mathrm{Sin}[0.218274\,L0])}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & \frac{587695.\,e^{0.218274\,L0}\,(-1+e^{0.436549\,L0})\,\mathrm{Sin}[0.218274\,L0]}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & \frac{64139.4}{1-4\,e^{0.436549}} \\
-\frac{587695.\,e^{0.218274\,L0}\,(-1+e^{0.436549\,L0})\,\mathrm{Sin}[0.218274\,L0]}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & -\frac{1.34623\times10^{6}\,e^{0.218274\,L0}\,((-1+e^{0.436549\,L0})\,\mathrm{Cos}[0.218274\,L0]-(1+e^{0.436549\,L0})\,\mathrm{Sin}[0.218274\,L0])}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & \frac{146924.}{1-4\,e^{0.436549}} \\
0 & 0 & \\
0 & 0 & \\
0 & 0 & \\
0 & 0 & \\
0 & 0 & \\
0 & 0 &
\end{array}\right.$$

Out[70]//MatrixForm=

$$\begin{pmatrix}
-\frac{74.4475\,(1+e^{0.436549\,L0}-2\,e^{0.218274\,L0}\,\mathrm{Cos}[0.218274\,L0])}{-1+e^{0.436549\,L0}+2\,e^{0.218274\,L0}\,\mathrm{Sin}[0.218274\,L0]} \\
-\frac{170.537\,(1-e^{0.436549\,L0}+2\,e^{0.218274\,L0}\,\mathrm{Sin}[0.218274\,L0])}{-1+e^{0.436549\,L0}+2\,e^{0.218274\,L0}\,\mathrm{Sin}[0.218274\,L0]} \\
-\frac{74.4475\,(1+e^{0.436549\,L0}-2\,e^{0.218274\,L0}\,\mathrm{Cos}[0.218274\,L0])}{-1+e^{0.436549\,L0}+2\,e^{0.218274\,L0}\,\mathrm{Sin}[0.218274\,L0]}-\frac{74.4475\,(1+e^{0.436549\,L1}-2\,e^{0.218274\,L1}\,\mathrm{Cos}[0.218274\,L1])}{-1+e^{0.436549\,L1}+2\,e^{0.218274\,L1}\,\mathrm{Sin}[0.218274\,L1]} \\
-\frac{170.537\,(-1+e^{0.436549\,L0}-2\,e^{0.218274\,L0}\,\mathrm{Sin}[0.218274\,L0])}{-1+e^{0.436549\,L0}+2\,e^{0.218274\,L0}\,\mathrm{Sin}[0.218274\,L0]}-\frac{170.537\,(1-e^{0.436549\,L1}+2\,e^{0.218274\,L1}\,\mathrm{Sin}[0.218274\,L1])}{-1+e^{0.436549\,L1}+2\,e^{0.218274\,L1}\,\mathrm{Sin}[0.218274\,L1]} \\
-\frac{74.4475\,(1+e^{0.436549\,L1}-2\,e^{0.218274\,L1}\,\mathrm{Cos}[0.218274\,L1])}{-1+e^{0.436549\,L1}+2\,e^{0.218274\,L1}\,\mathrm{Sin}[0.218274\,L1]}-\frac{7158.82\,\mathrm{Alpha}^3\,(1+e^{2\,\mathrm{Alpha}\,L2}-2\,e^{\mathrm{Alpha}\,L2}\,\mathrm{Cos}[\mathrm{Alpha}\,L2])}{-1+e^{2\,\mathrm{Alpha}\,L2}+2\,e^{\mathrm{Alpha}\,L2}\,\mathrm{Sin}[\mathrm{Alpha}\,L2]} \\
-\frac{170.537\,(-1+e^{0.436549\,L1}-2\,e^{0.218274\,L1}\,\mathrm{Sin}[0.218274\,L1])}{-1+e^{0.436549\,L1}+2\,e^{0.218274\,L1}\,\mathrm{Sin}[0.218274\,L1]}-\frac{3579.41\,\mathrm{Alpha}^2\,(1-e^{2\,\mathrm{Alpha}\,L2}+2\,e^{\mathrm{Alpha}\,L2}\,\mathrm{Sin}[\mathrm{Alpha}\,L2])}{-1+e^{2\,\mathrm{Alpha}\,L2}+2\,e^{\mathrm{Alpha}\,L2}\,\mathrm{Sin}[\mathrm{Alpha}\,L2]} \\
-\frac{7158.82\,\mathrm{Alpha}^3\,(1+e^{2\,\mathrm{Alpha}\,L2}-2\,e^{\mathrm{Alpha}\,L2}\,\mathrm{Cos}[\mathrm{Alpha}\,L2])}{-1+e^{2\,\mathrm{Alpha}\,L2}+2\,e^{\mathrm{Alpha}\,L2}\,\mathrm{Sin}[\mathrm{Alpha}\,L2]} \\
-\frac{3579.41\,\mathrm{Alpha}^2\,(-1+e^{2\,\mathrm{Alpha}\,L2}-2\,e^{\mathrm{Alpha}\,L2}\,\mathrm{Sin}[\mathrm{Alpha}\,L2])}{-1+e^{2\,\mathrm{Alpha}\,L2}+2\,e^{\mathrm{Alpha}\,L2}\,\mathrm{Sin}[\mathrm{Alpha}\,L2]} \\
0 \\
0
\end{pmatrix}$$

■ Terzo Tratto: espressioni numeriche

In[71]:=

```
Alpha01 = (k / (4 EI)) ^ .25
MatK03 = MatK /. {L → 5.9, Alpha → Alpha01};
MatrixForm[MatK03]
VectF003 = VectF0 /. {L → 5.9, Alpha → Alpha01};
MatrixForm[VectF003]
```

Out[71]= 0.218274

Out[73]//MatrixForm=

$$\begin{pmatrix}
120446. & -290894. & -79763.8 & -251063. \\
-290894. & 1.07224\times10^{6} & 251063. & 502626. \\
-79763.8 & 251063. & 120446. & 290894. \\
-251063. & 502626. & 290894. & 1.07224\times10^{6}
\end{pmatrix}$$

Out[75]//MatrixForm=

$$\begin{pmatrix}
-47.2206 \\
46.2323 \\
-47.2206 \\
-46.2323
\end{pmatrix}$$

In[76]:=
```
MatrixForm[MatKGlob] /. {L0 → 1, Alpha0 → Alpha01, L1 → 4.6, Alpha1 → Alpha01, L2 → 5.9, Alpha → Alpha01}
MatrixForm[VectF0Glob] /. {L0 → 1, Alpha0 → Alpha01, L1 → 4.6, Alpha1 → Alpha01, L2 → 5.9, Alpha → Alpha01}
```

Out[76]//MatrixForm=

$$\begin{pmatrix} 1.8508\times10^{7} & -9.25213\times10^{6} & -1.8501\times10^{7} & -9.25097\times10^{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ -9.25213\times10^{6} & 6.16773\times10^{6} & 9.25097\times10^{6} & 3.0837\times10^{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1.8501\times10^{7} & 9.25097\times10^{6} & 1.87219\times10^{7} & 8.7995\times10^{6} & -181898. & -428129. & 0 & 0 & 0 & 0 \\ -9.25097\times10^{6} & 3.0837\times10^{6} & 8.7995\times10^{6} & 7.5214\times10^{6} & 428129. & 660744. & 0 & 0 & 0 & 0 \\ 0 & 0 & -181898. & 428129. & 334363. & 161744. & -79763.8 & -251063. & 0 & 0 \\ 0 & 0 & -428129. & 660744. & 161744. & 2.42591\times10^{6} & 251063. & 502626. & 0 & 0 \\ 0 & 0 & 0 & 0 & -79763.8 & 251063. & 120446. & 290894. & 0 & 0 \\ 0 & 0 & 0 & 0 & -251063. & 502626. & 290894. & 1.07224\times10^{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Out[77]//MatrixForm=

$$\begin{pmatrix} -8.1249 \\ 1.35414 \\ -45.2906 \\ 27.0937 \\ -84.3863 \\ 17.7845 \\ -47.2206 \\ -46.2323 \\ 0 \\ 0 \end{pmatrix}$$

■ Assemblaggio Quarto Tratto

In[78]:=
```
Nodo1 = 4;
Nodo2 = 5;
```

In[80]:=
```
L = L3;
```

```
In[81]:= For[i = 1, i <= 2, For[j = 1, j <= 2,
           MatKGlob[[2 (Nodo1 - 1) + i, 2 (Nodo1 - 1) + j]] = MatKGlob[[2 (Nodo1 - 1) + i, 2 (Nodo1 - 1) + j]] + MatK[[i, j]] /. l → L; j++]; i++];
         For[i = 1, i <= 2, For[j = 1, j <= 2, MatKGlob[[2 (Nodo2 - 1) + i, 2 (Nodo1 - 1) + j]] =
             MatKGlob[[2 (Nodo2 - 1) + i, 2 (Nodo1 - 1) + j]] + MatK[[2 + i, j]] /. l → L; j++]; i++];
         For[i = 1, i <= 2, For[j = 1, j <= 2, MatKGlob[[2 (Nodo1 - 1) + i, 2 (Nodo2 - 1) + j]] =
             MatKGlob[[2 (Nodo1 - 1) + i, 2 (Nodo2 - 1) + j]] + MatK[[i, 2 + j]] /. l → L; j++]; i++];
         For[i = 1, i <= 2, For[j = 1, j <= 2, MatKGlob[[2 (Nodo2 - 1) + i, 2 (Nodo2 - 1) + j]] =
             MatKGlob[[2 (Nodo2 - 1) + i, 2 (Nodo2 - 1) + j]] + MatK[[2 + i, 2 + j]] /. l → L; j++]; i++];
         For[j = 1, j <= 2, VectF0Glob[[2 (Nodo1 - 1) + j]] = VectF0Glob[[2 (Nodo1 - 1) + j]] + VectF0[[j]] /. l → L; j++];
         For[j = 1, j <= 2, VectF0Glob[[2 (Nodo2 - 1) + j]] = VectF0Glob[[2 (Nodo2 - 1) + j]] + VectF0[[2 + j]] /. l → L; j++];
         MatrixForm[MatKGlob]
         MatrixForm[VectF0Glob]
```

Out[87]//MatrixForm=

$$\begin{pmatrix}
\frac{64139.4\,(-1+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Sin}[0.436549\,L0])}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & -\frac{146924.\,(1+e^{0.873098\,L0}-2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0])}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & \\
-\frac{146924.\,(1+e^{0.873098\,L0}-2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0])}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & \frac{673115.\,(-1+e^{0.873098\,L0}-2\,e^{0.436549\,L0}\,\mathrm{Sin}[0.436549\,L0])}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & \\
-\frac{128279.\,e^{0.218274\,L0}\,((-1+e^{0.436549\,L0})\,\mathrm{Cos}[0.218274\,L0]+(1+e^{0.436549\,L0})\,\mathrm{Sin}[0.218274\,L0])}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & \frac{587695.\,e^{0.218274\,L0}\,(-1+e^{0.436549\,L0})\,\mathrm{Sin}[0.218274\,L0]}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & \frac{64139.4}{1-4\,e^{0.436549}} \\
-\frac{587695.\,e^{0.218274\,L0}\,(-1+e^{0.436549\,L0})\,\mathrm{Sin}[0.218274\,L0]}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & -\frac{1.34623\times10^{6}\,e^{0.218274\,L0}\,((-1+e^{0.436549\,L0})\,\mathrm{Cos}[0.218274\,L0]-(1+e^{0.436549\,L0})\,\mathrm{Sin}[0.218274\,L0])}{1-4\,e^{0.436549\,L0}+e^{0.873098\,L0}+2\,e^{0.436549\,L0}\,\mathrm{Cos}[0.436549\,L0]} & \frac{146924.}{1-4\,e^{0.436549}} \\
0 & 0 & \\
0 & 0 & \\
0 & 0 & \\
0 & 0 & \\
0 & 0 & \\
0 & 0 &
\end{pmatrix}$$

Out[88]//MatrixForm=

$$\begin{pmatrix}
-\frac{74.4475\,(1+e^{0.436549\,\mathrm{L0}}-2\,e^{0.218274\,\mathrm{L0}}\,\mathrm{Cos}[0.218274\,\mathrm{L0}])}{-1+e^{0.436549\,\mathrm{L0}}+2\,e^{0.218274\,\mathrm{L0}}\,\mathrm{Sin}[0.218274\,\mathrm{L0}]} \\
-\frac{170.537\,(1-e^{0.436549\,\mathrm{L0}}+2\,e^{0.218274\,\mathrm{L0}}\,\mathrm{Sin}[0.218274\,\mathrm{L0}])}{-1+e^{0.436549\,\mathrm{L0}}+2\,e^{0.218274\,\mathrm{L0}}\,\mathrm{Sin}[0.218274\,\mathrm{L0}]} \\
-\frac{74.4475\,(1+e^{0.436549\,\mathrm{L0}}-2\,e^{0.218274\,\mathrm{L0}}\,\mathrm{Cos}[0.218274\,\mathrm{L0}])}{-1+e^{0.436549\,\mathrm{L0}}+2\,e^{0.218274\,\mathrm{L0}}\,\mathrm{Sin}[0.218274\,\mathrm{L0}]}-\frac{74.4475\,(1+e^{0.436549\,\mathrm{L1}}-2\,e^{0.218274\,\mathrm{L1}}\,\mathrm{Cos}[0.218274\,\mathrm{L1}])}{-1+e^{0.436549\,\mathrm{L1}}+2\,e^{0.218274\,\mathrm{L1}}\,\mathrm{Sin}[0.218274\,\mathrm{L1}]} \\
-\frac{170.537\,(-1+e^{0.436549\,\mathrm{L0}}-2\,e^{0.218274\,\mathrm{L0}}\,\mathrm{Sin}[0.218274\,\mathrm{L0}])}{-1+e^{0.436549\,\mathrm{L0}}+2\,e^{0.218274\,\mathrm{L0}}\,\mathrm{Sin}[0.218274\,\mathrm{L0}]}-\frac{170.537\,(1-e^{0.436549\,\mathrm{L1}}+2\,e^{0.218274\,\mathrm{L1}}\,\mathrm{Sin}[0.218274\,\mathrm{L1}])}{-1+e^{0.436549\,\mathrm{L1}}+2\,e^{0.218274\,\mathrm{L1}}\,\mathrm{Sin}[0.218274\,\mathrm{L1}]} \\
-\frac{74.4475\,(1+e^{0.436549\,\mathrm{L1}}-2\,e^{0.218274\,\mathrm{L1}}\,\mathrm{Cos}[0.218274\,\mathrm{L1}])}{-1+e^{0.436549\,\mathrm{L1}}+2\,e^{0.218274\,\mathrm{L1}}\,\mathrm{Sin}[0.218274\,\mathrm{L1}]}-\frac{7158.82\,\mathrm{Alpha}^3\,(1+e^{2\,\mathrm{Alpha\,L2}}-2\,e^{\mathrm{Alpha\,L2}}\,\mathrm{Cos}[\mathrm{Alpha\,L2}])}{-1+e^{2\,\mathrm{Alpha\,L2}}+2\,e^{\mathrm{Alpha\,L2}}\,\mathrm{Sin}[\mathrm{Alpha\,L2}]} \\
-\frac{170.537\,(-1+e^{0.436549\,\mathrm{L1}}-2\,e^{0.218274\,\mathrm{L1}}\,\mathrm{Sin}[0.218274\,\mathrm{L1}])}{-1+e^{0.436549\,\mathrm{L1}}+2\,e^{0.218274\,\mathrm{L1}}\,\mathrm{Sin}[0.218274\,\mathrm{L1}]}-\frac{3579.41\,\mathrm{Alpha}^2\,(1-e^{2\,\mathrm{Alpha\,L2}}+2\,e^{\mathrm{Alpha\,L2}}\,\mathrm{Sin}[\mathrm{Alpha\,L2}])}{-1+e^{2\,\mathrm{Alpha\,L2}}+2\,e^{\mathrm{Alpha\,L2}}\,\mathrm{Sin}[\mathrm{Alpha\,L2}]} \\
-\frac{7158.82\,\mathrm{Alpha}^3\,(1+e^{2\,\mathrm{Alpha\,L2}}-2\,e^{\mathrm{Alpha\,L2}}\,\mathrm{Cos}[\mathrm{Alpha\,L2}])}{-1+e^{2\,\mathrm{Alpha\,L2}}+2\,e^{\mathrm{Alpha\,L2}}\,\mathrm{Sin}[\mathrm{Alpha\,L2}]}-\frac{7158.82\,\mathrm{Alpha}^3\,(1+e^{2\,\mathrm{Alpha\,L3}}-2\,e^{\mathrm{Alpha\,L3}}\,\mathrm{Cos}[\mathrm{Alpha\,L3}])}{-1+e^{2\,\mathrm{Alpha\,L3}}+2\,e^{\mathrm{Alpha\,L3}}\,\mathrm{Sin}[\mathrm{Alpha\,L3}]} \\
-\frac{3579.41\,\mathrm{Alpha}^2\,(-1+e^{2\,\mathrm{Alpha\,L2}}-2\,e^{\mathrm{Alpha\,L2}}\,\mathrm{Sin}[\mathrm{Alpha\,L2}])}{-1+e^{2\,\mathrm{Alpha\,L2}}+2\,e^{\mathrm{Alpha\,L2}}\,\mathrm{Sin}[\mathrm{Alpha\,L2}]}-\frac{3579.41\,\mathrm{Alpha}^2\,(1-e^{2\,\mathrm{Alpha\,L3}}+2\,e^{\mathrm{Alpha\,L3}}\,\mathrm{Sin}[\mathrm{Alpha\,L3}])}{-1+e^{2\,\mathrm{Alpha\,L3}}+2\,e^{\mathrm{Alpha\,L3}}\,\mathrm{Sin}[\mathrm{Alpha\,L3}]} \\
-\frac{7158.82\,\mathrm{Alpha}^3\,(1+e^{2\,\mathrm{Alpha\,L3}}-2\,e^{\mathrm{Alpha\,L3}}\,\mathrm{Cos}[\mathrm{Alpha\,L3}])}{-1+e^{2\,\mathrm{Alpha\,L3}}+2\,e^{\mathrm{Alpha\,L3}}\,\mathrm{Sin}[\mathrm{Alpha\,L3}]} \\
-\frac{3579.41\,\mathrm{Alpha}^2\,(-1+e^{2\,\mathrm{Alpha\,L3}}-2\,e^{\mathrm{Alpha\,L3}}\,\mathrm{Sin}[\mathrm{Alpha\,L3}])}{-1+e^{2\,\mathrm{Alpha\,L3}}+2\,e^{\mathrm{Alpha\,L3}}\,\mathrm{Sin}[\mathrm{Alpha\,L3}]}
\end{pmatrix}$$

■ Quarto Tratto: espressioni numeriche

In[89]:=
```
Alpha01 = (k / (4 EI)) ^.25
MatK04 = MatK /. {L → 1, Alpha → Alpha01};
MatrixForm[MatK04]
VectF004 = VectF0 /. {L → 1, Alpha → Alpha01};
MatrixForm[VectF004]
```

Out[89]= 0.218274

Out[91]//MatrixForm=

$$\begin{pmatrix} 1.8508\times10^{7} & -9.25213\times10^{6} & -1.8501\times10^{7} & -9.25097\times10^{6} \\ -9.25213\times10^{6} & 6.16773\times10^{6} & 9.25097\times10^{6} & 3.0837\times10^{6} \\ -1.8501\times10^{7} & 9.25097\times10^{6} & 1.8508\times10^{7} & 9.25213\times10^{6} \\ -9.25097\times10^{6} & 3.0837\times10^{6} & 9.25213\times10^{6} & 6.16773\times10^{6} \end{pmatrix}$$

Out[93]//MatrixForm=

$$\begin{pmatrix} -8.1249 \\ 1.35414 \\ -8.1249 \\ -1.35414 \end{pmatrix}$$

In[94]:=
```
MatrixForm[MatKGlob] /. {L0 → 1, Alpha0 → Alpha01, L1 → 4.6, Alpha1 → Alpha01, L2 → 5.9, Alpha → Alpha01, L3 → 1}
MatrixForm[VectF0Glob] /. {L0 → 1, Alpha0 → Alpha01, L1 → 4.6, Alpha1 → Alpha01, L2 → 5.9, Alpha → Alpha01, L3 → 1}
```

Out[94]//MatrixForm=

1.8508×10^{7}	-9.25213×10^{6}	-1.8501×10^{7}	-9.25097×10^{6}	0	0	0	0	0	0
-9.25213×10^{6}	6.16773×10^{6}	9.25097×10^{6}	3.0837×10^{6}	0	0	0	0	0	0
-1.8501×10^{7}	9.25097×10^{6}	1.87219×10^{7}	8.7995×10^{6}	-181898.	-428129.	0	0	0	0
-9.25097×10^{6}	3.0837×10^{6}	8.7995×10^{6}	7.5214×10^{6}	428129.	660744.	0	0	0	0
0	0	-181898.	428129.	334363.	161744.	-79763.8	-251063.	0	0
0	0	-428129.	660744.	161744.	2.42591×10^{6}	251063.	502626.	0	0
0	0	0	0	-79763.8	251063.	1.86284×10^{7}	-8.96124×10^{6}	-1.8501×10^{7}	$-9.25097\times$
0	0	0	0	-251063.	502626.	-8.96124×10^{6}	7.23997×10^{6}	9.25097×10^{6}	3.0837×10
0	0	0	0	0	0	-1.8501×10^{7}	9.25097×10^{6}	1.8508×10^{7}	$9.25213\times$
0	0	0	0	0	0	-9.25097×10^{6}	3.0837×10^{6}	9.25213×10^{6}	$6.16773\times$

Out[95]//MatrixForm=

-8.1249
1.35414
-45.2906
27.0937
-84.3863
17.7845
-55.3455
-44.8781
-8.1249
-1.35414

■ Vettore delle Forze Nodali applicate

In[96]:=
```
VectFGlob = {0, 0, N4Ed, M4Ed, N5Ed, M5Ed, N6Ed, M6Ed, 0, 0}
```

■ Vettore delle Forze Nodali: valori numerici

```
In[97]:= VectFGlobNum = {0.00, 0.00, 216.64, −323.49, 714.21, −471.00, 524.10, −382.23, 0.00, 0.00}

Out[97]= {0., 0., 216.64, -323.49, 714.21, -471., 524.1, -382.23, 0., 0.}

In[98]:= MatKGlobNum = MatKGlob /. {L0 → 1, Alpha0 → Alpha01, L1 → 4.6, Alpha1 → Alpha01, L2 → 5.9, Alpha → Alpha01, L3 → 1};
         VectF0GlobNum = VectF0Glob /. {L0 → 1, Alpha0 → Alpha01, L1 → 4.6, Alpha1 → Alpha01, L2 → 5.9, Alpha → Alpha01, L3 → 1};
         MatrixForm[MatKGlobNum]
         MatrixForm[VectF0GlobNum]
```

Out[100]//MatrixForm=

1.8508×10^{7}	-9.25213×10^{6}	-1.8501×10^{7}	-9.25097×10^{6}	0	0	0	0	0	0
-9.25213×10^{6}	6.16773×10^{6}	9.25097×10^{6}	3.0837×10^{6}	0	0	0	0	0	0
-1.8501×10^{7}	9.25097×10^{6}	1.87219×10^{7}	8.7995×10^{6}	-181898.	-428129.	0	0	0	0
-9.25097×10^{6}	3.0837×10^{6}	8.7995×10^{6}	7.5214×10^{6}	428129.	660744.	0	0	0	0
0	0	-181898.	428129.	334363.	161744.	-79763.8	-251063.	0	0
0	0	-428129.	660744.	161744.	2.42591×10^{6}	251063.	502626.	0	0
0	0	0	0	-79763.8	251063.	1.86284×10^{7}	-8.96124×10^{6}	-1.8501×10^{7}	-9.25097×
0	0	0	0	-251063.	502626.	-8.96124×10^{6}	7.23997×10^{6}	9.25097×10^{6}	3.0837×10
0	0	0	0	0	0	-1.8501×10^{7}	9.25097×10^{6}	1.8508×10^{7}	9.25213×
0	0	0	0	0	0	-9.25097×10^{6}	3.0837×10^{6}	9.25213×10^{6}	6.16773×

Out[101]//MatrixForm=

-8.1249
1.35414
-45.2906
27.0937
-84.3863
17.7845
-55.3455
-44.8781
-8.1249
-1.35414

■ Soluzione del sistema: Calcolo degli spostamenti globali

```
In[102]:= sNum = Inverse[MatKGlobNum] . (VectFGlobNum - VectF0GlobNum)

Out[102]= {0.00276883, -0.00130421, 0.00407233, -0.00130128, 0.00900906, -0.000936025, 0.0152956, -0.00181466, 0.0170927, -0.00179122}
```

■ Post-Processing: calcolo delle espressioni della linea elastica e delle caratteristiche della sollecitazione

■ Tratto 1

```
In[165]:= Nodo1 = 1;
          Nodo2 = 2;

In[167]:= L0 = 1;

In[168]:= w1 = Funzioni.VectA + q / k /. Alpha → Alpha01

Out[168]= 0.00116071 + A2 ⅇ^(-0.218274 z) Cos[0.218274 z] + A4 ⅇ^(0.218274 z) Cos[0.218274 z] + A1 ⅇ^(-0.218274 z) Sin[0.218274 z] + A3 ⅇ^(0.218274 z) Sin[0.218274 z]

In[169]:= s1 = Table[sNum[[i]], {i, 2 Nodo1 - 1, 2 Nodo2}]

Out[169]= {0.00276883, -0.00130421, 0.00407233, -0.00130128}

In[170]:= Sistema1 = {(w1 /. z → 0) - s1[[1]], (-D[w1, z] /. z → 0) - s1[[2]], (w1 /. z → L0) - s1[[3]], (-D[w1, z] /. z → L0) - s1[[4]]}

Out[170]= {-0.00160811 + A2 + A4, 0.00130421 - 0.218274 A1 + 0.218274 A2 - 0.218274 A3 - 0.218274 A4,
           -0.00291162 + 0.174082 A1 + 0.78483 A2 + 0.269367 A3 + 1.21441 A4, 0.00130128 - 0.133311 A1 + 0.209306 A2 - 0.323871 A3 - 0.206279 A4}

In[171]:= Sol01 = Solve[Sistema1 == 0, VectA] // Flatten

Out[171]= {A1 → 0.00149377, A2 → -0.000689717, A3 → 0.00149377, A4 → 0.00229783}
```

```
In[172]:= VectA01 = VectA /. Sol01

Out[172]= {0.00149377, -0.000689717, 0.00149377, 0.00229783}

In[173]:= w1Sol = Funzioni.VectA01 + q / k /. Alpha → Alpha01
```

Out[173]= $0.00116071 - 0.000689717\, e^{-0.218274 z} \mathrm{Cos}[0.218274 z] + 0.00229783\, e^{0.218274 z} \mathrm{Cos}[0.218274 z] + 0.00149377\, e^{-0.218274 z} \mathrm{Sin}[0.218274 z] + 0.00149377\, e^{0.218274 z} \mathrm{Sin}[0.218274 z]$

■ Tratto 2

```
In[174]:= Nodo1 = 2;
          Nodo2 = 3;

In[176]:= L1 = 4.6;

In[177]:= w2 = Funzioni.VectA + q / k /. Alpha → Alpha01
```

Out[177]= $0.00116071 + A2\, e^{-0.218274 z} \mathrm{Cos}[0.218274 z] + A4\, e^{0.218274 z} \mathrm{Cos}[0.218274 z] + A1\, e^{-0.218274 z} \mathrm{Sin}[0.218274 z] + A3\, e^{0.218274 z} \mathrm{Sin}[0.218274 z]$

```
In[178]:= s2 = Table[sNum[[i]], {i, 2 Nodo1 - 1, 2 Nodo2}]

Out[178]= {0.00407233, -0.00130128, 0.00900906, -0.000936025}

In[179]:= Sistema2 = {(w2 /. z → 0) - s2[[1]], (-D[w2, z] /. z → 0) - s2[[2]], (w2 /. z → L1) - s2[[3]], (-D[w2, z] /. z → L1) - s2[[4]]}

Out[179]= {-0.00291162 + A2 + A4, 0.00130128 - 0.218274 A1 + 0.218274 A2 - 0.218274 A3 - 0.218274 A4,
           -0.00784835 + 0.309106 A1 + 0.196706 A2 + 2.30264 A3 + 1.46533 A4, 0.000936025 + 0.0245341 A1 + 0.110406 A2 - 0.822451 A3 + 0.182763 A4}

In[180]:= Sol02 = Solve[Sistema2 == 0, VectA] // Flatten

Out[180]= {A1 → 0.00408212, A2 → 0.00140755, A3 → 0.00178304, A4 → 0.00150407}

In[181]:= VectA02 = VectA /. Sol02

Out[181]= {0.00408212, 0.00140755, 0.00178304, 0.00150407}
```

```
In[182]:= w2Sol = Funzioni.VectA02 + q / k /. Alpha → Alpha01

Out[182]= 0.00116071 + 0.00140755 e^(-0.218274 z) Cos[0.218274 z] + 0.00150407 e^(0.218274 z) Cos[0.218274 z] +
           0.00408212 e^(-0.218274 z) Sin[0.218274 z] + 0.00178304 e^(0.218274 z) Sin[0.218274 z]
```

■ Tratto 3

```
In[121]:= Nodo1 = 3;
          Nodo2 = 4;

In[123]:= L2 = 5.9;

In[124]:= w3 = Funzioni.VectA + q / k /. Alpha → Alpha01

Out[124]= 0.00116071 + A2 e^(-0.218274 z) Cos[0.218274 z] + A4 e^(0.218274 z) Cos[0.218274 z] + A1 e^(-0.218274 z) Sin[0.218274 z] + A3 e^(0.218274 z) Sin[0.218274 z]

In[125]:= s3 = Table[sNum[[i]], {i, 2 Nodo1 - 1, 2 Nodo2}]

Out[125]= {0.00900906, -0.000936025, 0.0152956, -0.00181466}

In[126]:= Sistema3 = {(w3 /. z → 0) - s3[[1]], (-D[w3, z] /. z → 0) - s3[[2]], (w3 /. z → L2) - s3[[3]], (-D[w3, z] /. z → L2) - s3[[4]]}

Out[126]= {-0.00784835 + A2 + A4, 0.000936025 - 0.218274 A1 + 0.218274 A2 - 0.218274 A3 - 0.218274 A4,
           -0.0141349 + 0.2649 A1 + 0.0770276 A2 + 3.48071 A3 + 1.01212 A4, 0.00181466 + 0.0410077 A1 + 0.074634 A2 - 0.98067 A3 + 0.538829 A4}

In[127]:= Sol03 = Solve[Sistema3 == 0, VectA] // Flatten

Out[127]= {A1 → 0.0075384, A2 → 0.00710632, A3 → 0.00311419, A4 → 0.000742025}

In[128]:= VectA03 = VectA /. Sol03

Out[128]= {0.0075384, 0.00710632, 0.00311419, 0.000742025}

In[129]:= w3Sol = Funzioni.VectA03 + q / k /. Alpha → Alpha01

Out[129]= 0.00116071 + 0.00710632 e^(-0.218274 z) Cos[0.218274 z] + 0.000742025 e^(0.218274 z) Cos[0.218274 z] +
           0.0075384 e^(-0.218274 z) Sin[0.218274 z] + 0.00311419 e^(0.218274 z) Sin[0.218274 z]
```

■ Tratto 4

In[130]:=
```
Nodo1 = 4;
Nodo2 = 5;
```

In[132]:=
```
L3 = 1;
```

In[133]:=
```
w4 = Funzioni.VectA + q / k /. Alpha → Alpha01
```

Out[133]= $0.00116071 + A2\, e^{-0.218274\, z} \mathrm{Cos}[0.218274\, z] + A4\, e^{0.218274\, z} \mathrm{Cos}[0.218274\, z] + A1\, e^{-0.218274\, z} \mathrm{Sin}[0.218274\, z] + A3\, e^{0.218274\, z} \mathrm{Sin}[0.218274\, z]$

In[134]:=
```
s4 = Table[sNum[[i]], {i, 2 Nodo1 - 1, 2 Nodo2}]
```

Out[134]= {0.0152956, -0.00181466, 0.0170927, -0.00179122}

In[135]:=
```
Sistema4 = {(w4 /. z → 0) - s4[[1]], (-D[w4, z] /. z → 0) - s4[[2]], (w4 /. z → L0) - s4[[3]], (-D[w4, z] /. z → L0) - s4[[4]]}
```

Out[135]= {-0.0141349 + A2 + A4, 0.00181466 - 0.218274 A1 + 0.218274 A2 - 0.218274 A3 - 0.218274 A4,
-0.015932 + 0.174082 A1 + 0.78483 A2 + 0.269367 A3 + 1.21441 A4, 0.00179122 - 0.133311 A1 + 0.209306 A2 - 0.323871 A3 - 0.206279 A4}

In[136]:=
```
Sol04 = Solve[Sistema4 == 0, VectA] // Flatten
```

Out[136]= {A1 → 0.00408461, A2 → 0.00662994, A3 → 0.00335401, A4 → 0.00750495}

In[137]:=
```
VectA04 = VectA /. Sol04
```

Out[137]= {0.00408461, 0.00662994, 0.00335401, 0.00750495}

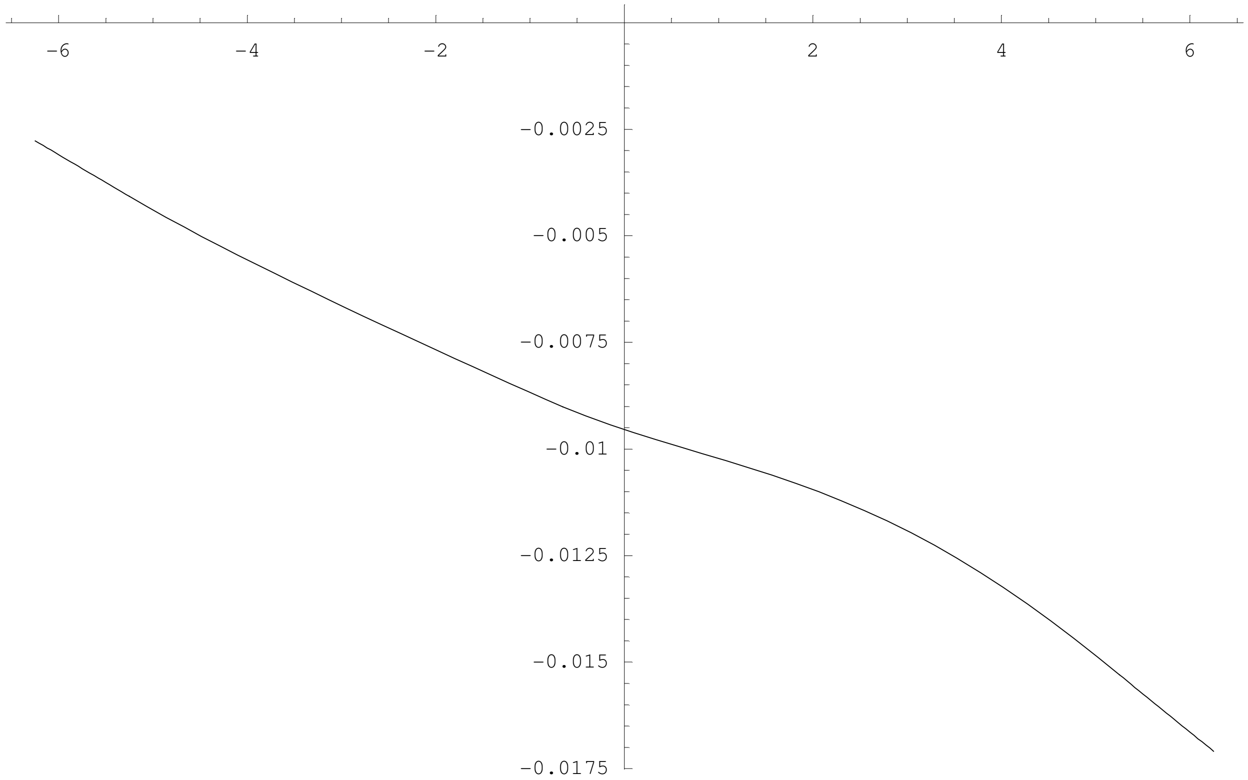
In[138]:=
```
w4Sol = Funzioni.VectA04 + q / k /. Alpha → Alpha01
```

Out[138]= $0.00116071 + 0.00662994\, e^{-0.218274\, z} \mathrm{Cos}[0.218274\, z] + 0.00750495\, e^{0.218274\, z} \mathrm{Cos}[0.218274\, z] + 0.00408461\, e^{-0.218274\, z} \mathrm{Sin}[0.218274\, z] + 0.00335401\, e^{0.218274\, z} \mathrm{Sin}[0.218274\, z]$

■ Grafici

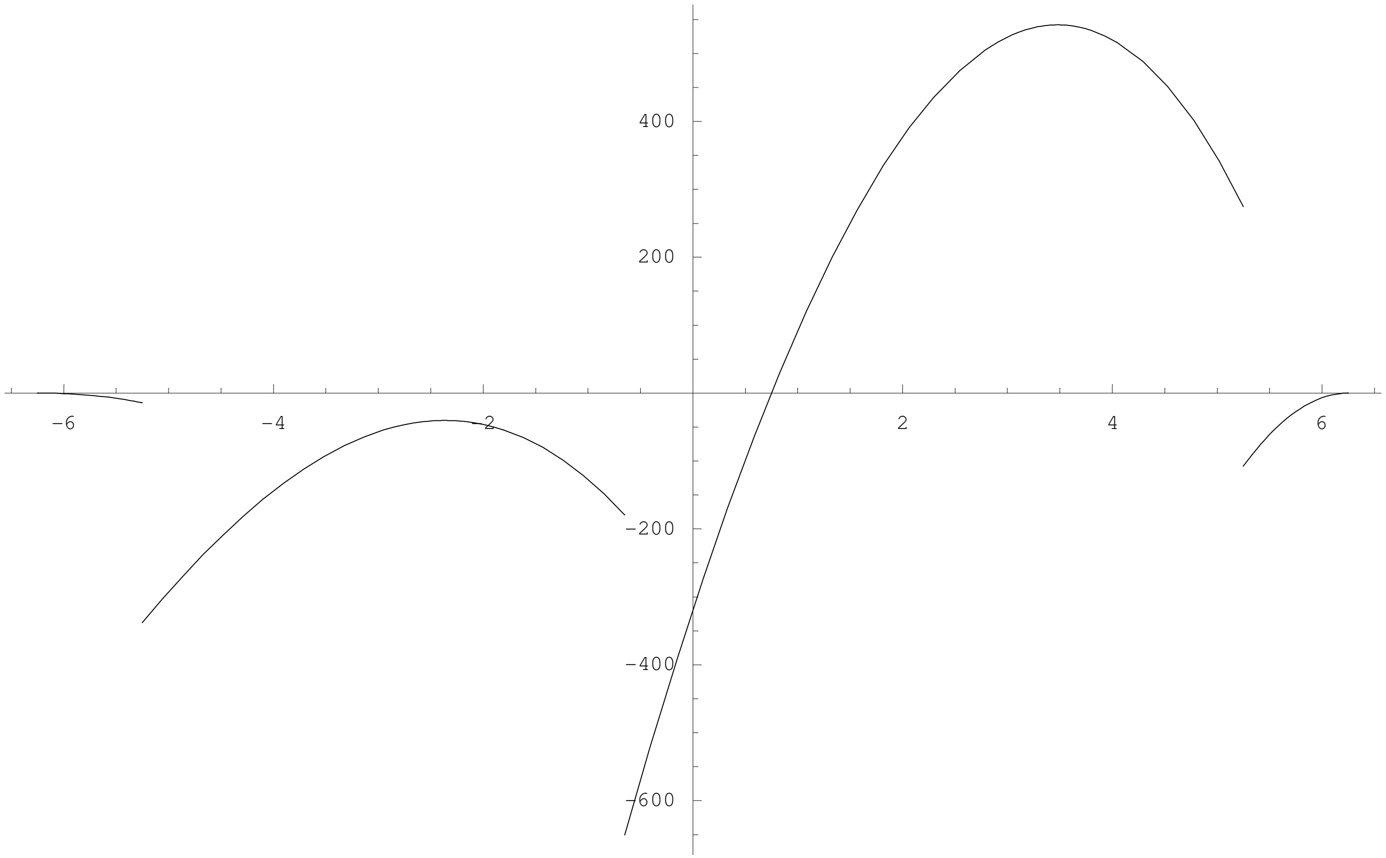
```
In[139]:= L0 = 1;
          L1 = 4.6;
          L2 = 5.9;
          L3 = 1;

In[143]:= Lm = L0 + L1 + L2 + L3;
```

```
In[144]:= Show[Plot[-w1Sol /. z → (zeta + Lm / 2), {zeta, -Lm / 2, -Lm / 2 + L0}],
    Plot[-w2Sol /. z → (zeta + Lm / 2 - L0), {zeta, -Lm / 2 + L0, -Lm / 2 + (L1 + L0)}],
    Plot[-w3Sol /. z → (zeta + Lm / 2 - L0 - L1), {zeta, -Lm / 2 + (L0 + L1), -Lm / 2 + (L0 + L1 + L2)}],
    Plot[-w4Sol /. z → (zeta + Lm / 2 - L0 - L1 - L2), {zeta, -Lm / 2 + (L0 + L1 + L2), -Lm / 2 + (L0 + L1 + L2 + L3)}]]
```



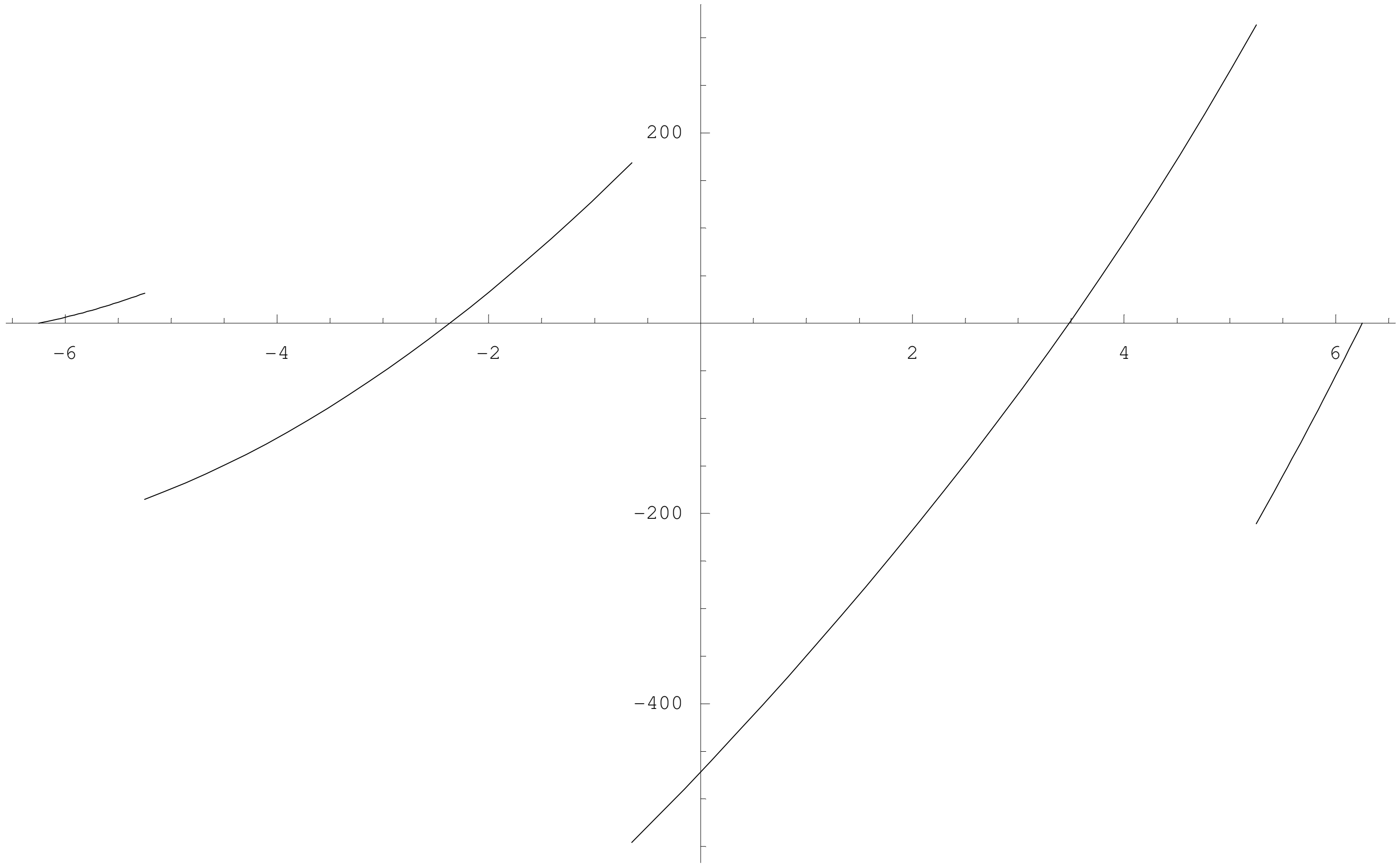

```
Out[144]= - Graphics -
```

```
In[145]:= Show[Plot[EI D[w1Sol, {z, 2}] /. z → (zeta + Lm / 2), {zeta, -Lm / 2, -Lm / 2 + L0}],
  Plot[EI D[w2Sol, {z, 2}] /. z → (zeta - L0 + Lm / 2), {zeta, -Lm / 2 + L0, -Lm / 2 + L1 + L0}],
  Plot[EI D[w3Sol, {z, 2}] /. z → (zeta - L0 - L1 + Lm / 2), {zeta, -Lm / 2 + L0 + L1, -Lm / 2 + L0 + L1 + L2}],
  Plot[EI D[w4Sol, {z, 2}] /. z → (zeta - L0 - L1 - L2 + Lm / 2), {zeta, -Lm / 2 + L0 + L1 + L2, -Lm / 2 + L0 + L1 + L2 + L3}]]
```




```
Out[145]= - Graphics -
```

```
In[146]:= Show[Plot[-EI D[w1Sol, {z, 3}] /. z → (zeta + Lm / 2), {zeta, -Lm / 2, -Lm / 2 + L0}],
  Plot[-EI D[w2Sol, {z, 3}] /. z → (zeta - L0 + Lm / 2), {zeta, -Lm / 2 + L0, -Lm / 2 + L1 + L0}],
  Plot[-EI D[w3Sol, {z, 3}] /. z → (zeta - L0 - L1 + Lm / 2), {zeta, -Lm / 2 + L0 + L1, -Lm / 2 + L0 + L1 + L2}],
  Plot[-EI D[w4Sol, {z, 3}] /. z → (zeta - L0 - L1 - L2 + Lm / 2), {zeta, -Lm / 2 + L0 + L1 + L2, -Lm / 2 + L0 + L1 + L2 + L3}]]
```




```
Out[146]= - Graphics -
```

■ Soluzione nell'ipotesi di trave rigida su suolo alla Winkler

```
In[147]:= VectFGlobNum = {0.00, 0.00, 216.64, −323.49, 714.21, −471.00, 524.10, −382.23, 0.00, 0.00}

Out[147]= {0., 0., 216.64, -323.49, 714.21, -471., 524.1, -382.23, 0., 0.}

In[148]:= Bm = 1.4;
          Lm = L0 + L1 + L2 + L3;

In[150]:= NEd = {VectFGlobNum[[3]], VectFGlobNum[[5]], VectFGlobNum[[7]]}
          MEd = {VectFGlobNum[[4]], VectFGlobNum[[6]], VectFGlobNum[[8]]}
          ei = {- (L0 + L1 + L2 + L3) / 2 + L0, - (L0 + L1 + L2 + L3) / 2 + L0 + L1, - (L0 + L1 + L2 + L3) / 2 + L0 + L1 + L2}
          MomG = Sum[NEd[[i]] ei[[i]], {i, 1, 3}] - Sum[MEd[[i]], {i, 1, 3}]
          Norm = Sum[NEd[[i]], {i, 1, 3}] + q (L0 + L1 + L2 + L3)
          Ecc = MomG / Norm

Out[150]= {216.64, 714.21, 524.1}

Out[151]= {-323.49, -471., -382.23}

Out[152]= {-5.25, -0.65, 5.25}

Out[153]= 2326.65

Out[154]= 1658.08

Out[155]= 1.40322
```

In[156]:=
```
wmax = (Norm / (Lm) + MomG / (Lm^2 / 6)) / k
wmin = (Norm / (Lm) - MomG / (Lm^2 / 6)) / k
wRigido = (wmax + wmin) / 2 + (wmax - wmin) / Lm zeta
```

Out[156]= 0.0158564

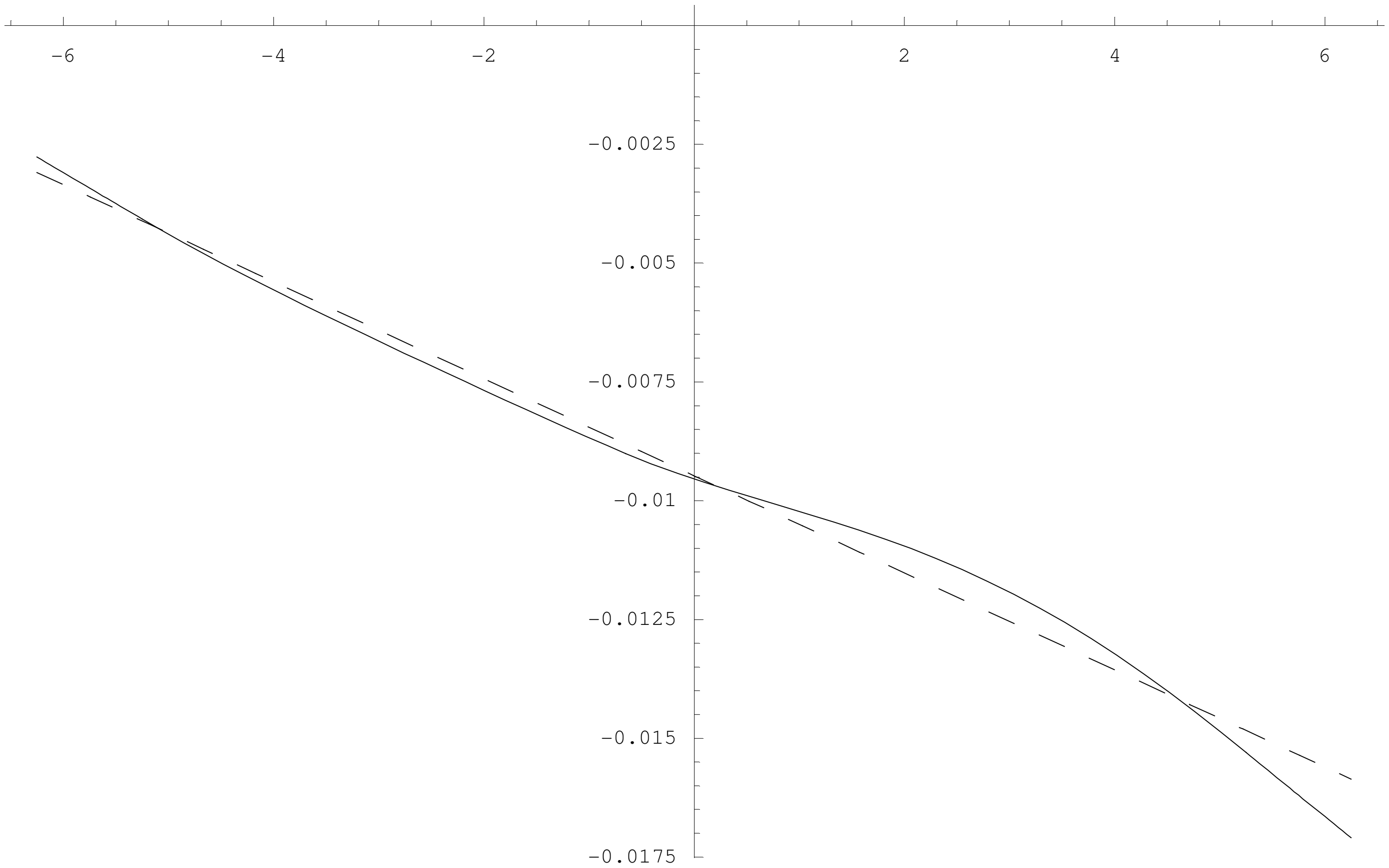
Out[157]= 0.00309305

Out[158]= 0.00947471 + 0.00102107 zeta

■ Confronti

■ Campo di spostamenti

```
In[159]:= Show[Plot[-w1Sol /. z → (zeta + Lm / 2), {zeta, -Lm / 2, -Lm / 2 + L0}],
  Plot[-w2Sol /. z → (zeta + Lm / 2 - L0), {zeta, -Lm / 2 + L0, -Lm / 2 + (L1 + L0)}],
  Plot[-w3Sol /. z → (zeta + Lm / 2 - L0 - L1), {zeta, -Lm / 2 + (L0 + L1), -Lm / 2 + (L0 + L1 + L2)}],
  Plot[-w4Sol /. z → (zeta + Lm / 2 - L0 - L1 - L2), {zeta, -Lm / 2 + (L0 + L1 + L2), -Lm / 2 + (L0 + L1 + L2 + L3)}],
  Plot[-wRigido, {zeta, -Lm / 2, Lm / 2}, PlotStyle -> Dashing[{0.02, 0.02}]]]
```

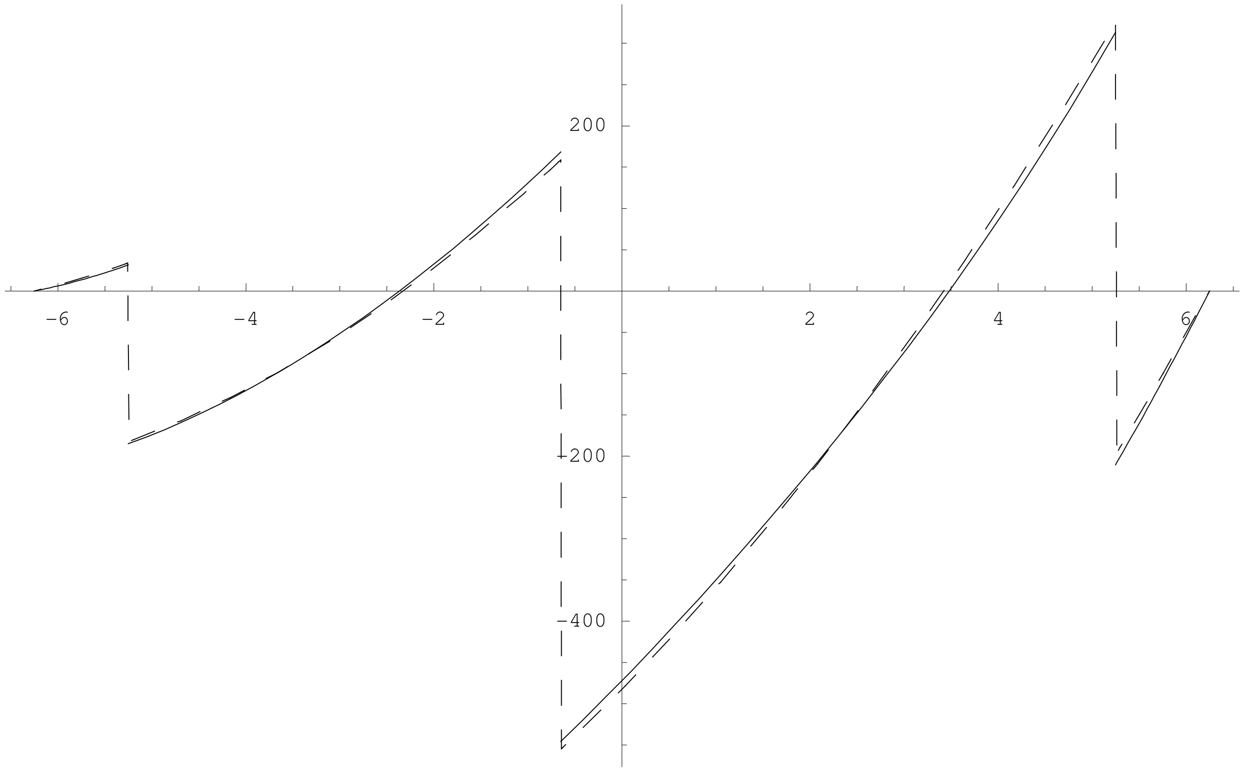


Out[159]= - Graphics -

■ Diagramma del Taglio

In[160]:=
```
TaglioRig[z_] := Integrate[k wRigido - q, {zeta, -Lm / 2, z}] -
   If[z ≤ -Lm / 2 + L0, 0, NEd[[1]]] - If[z ≤ -Lm / 2 + L0 + L1, 0, NEd[[2]]] - If[z ≤ -Lm / 2 + L0 + L1 + L2, 0, NEd[[3]]]
```

```
In[161]:= Show[Plot[-EI D[w1Sol, {z, 3}] /. z → (zeta + Lm / 2), {zeta, -Lm / 2, -Lm / 2 + L0}],
   Plot[-EI D[w2Sol, {z, 3}] /. z → (zeta - L0 + Lm / 2), {zeta, -Lm / 2 + L0, -Lm / 2 + L1 + L0}],
   Plot[-EI D[w3Sol, {z, 3}] /. z → (zeta - L0 - L1 + Lm / 2), {zeta, -Lm / 2 + L0 + L1, -Lm / 2 + L0 + L1 + L2}],
   Plot[-EI D[w4Sol, {z, 3}] /. z → (zeta - L0 - L1 - L2 + Lm / 2), {zeta, -Lm / 2 + L0 + L1 + L2, -Lm / 2 + L0 + L1 + L2 + L3}],
   Plot[TaglioRig[z], {z, -Lm / 2, Lm / 2}, PlotStyle -> Dashing[{0.02, 0.02}]]]
```

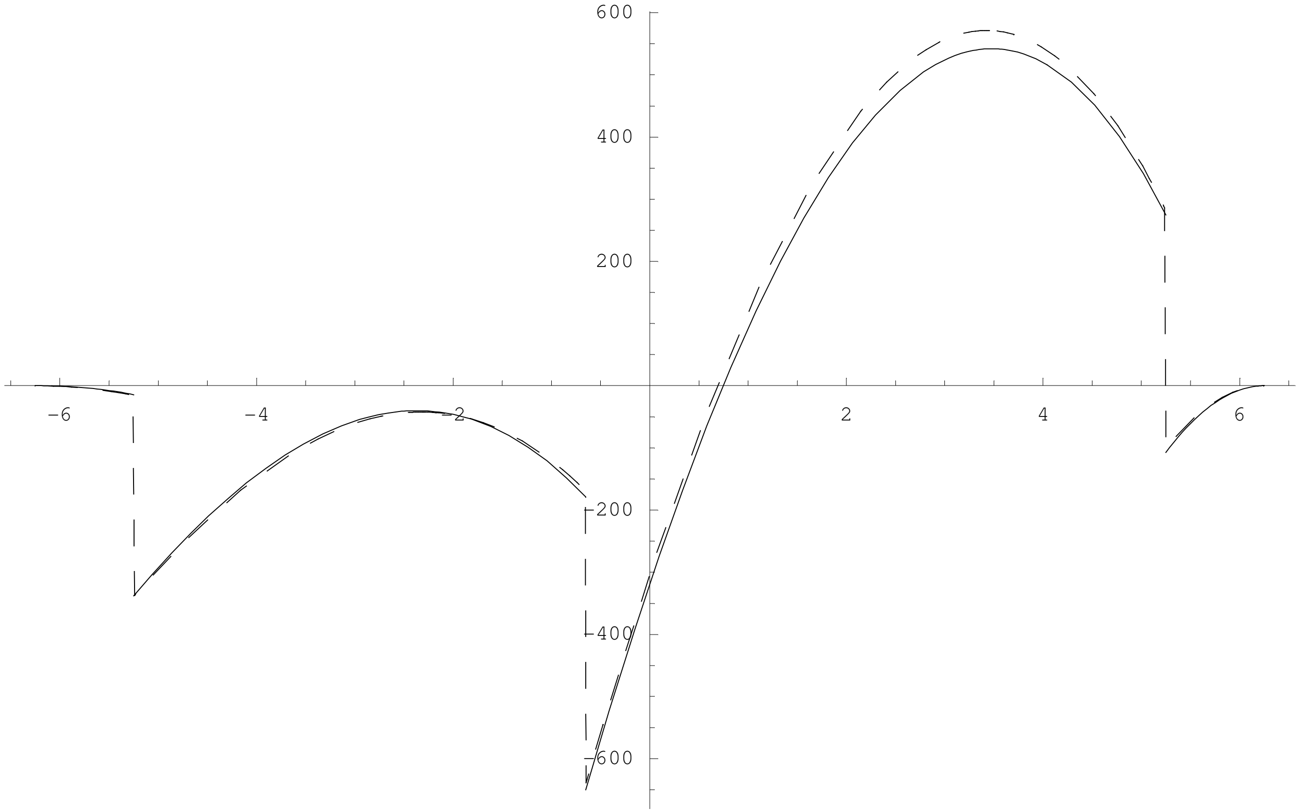



```
Out[161]= - Graphics -
```

■ Diagramma del Momento flettente

```
In[162]:= MomentoRig[z_] := Integrate[(k wRigido - q) (z - zeta), {zeta, -Lm / 2, z}] - If[z ≤ -Lm / 2 + L0, 0, MEd[[1]]] -
    If[z ≤ -Lm / 2 + L0 + L1, 0, MEd[[2]]] - If[z ≤ -Lm / 2 + L0 + L1 + L2, 0, MEd[[3]]] + If[z ≤ -Lm / 2 + L0, 0, NEd[[1]] (ei[[1]] - z)] +
    If[z ≤ -Lm / 2 + L0 + L1, 0, NEd[[2]] (ei[[2]] - z)] + If[z ≤ -Lm / 2 + L0 + L1 + L2, 0, NEd[[3]] (ei[[3]] - z)]

In[163]:=
```

```
In[164]:= Show[Plot[EI D[w1Sol, {z, 2}] /. z → (zeta + Lm / 2), {zeta, -Lm / 2, -Lm / 2 + L0}],
           Plot[EI D[w2Sol, {z, 2}] /. z → (zeta - L0 + Lm / 2), {zeta, -Lm / 2 + L0, -Lm / 2 + L1 + L0}],
           Plot[EI D[w3Sol, {z, 2}] /. z → (zeta - L0 - L1 + Lm / 2), {zeta, -Lm / 2 + L0 + L1, -Lm / 2 + L0 + L1 + L2}],
           Plot[EI D[w4Sol, {z, 2}] /. z → (zeta - L0 - L1 - L2 + Lm / 2), {zeta, -Lm / 2 + L0 + L1 + L2, -Lm / 2 + L0 + L1 + L2 + L3}],
           Plot[-MomentoRig[z], {z, -Lm / 2, Lm / 2}, PlotStyle -> Dashing[{0.02, 0.02}]]]
```




```
Out[164]= - Graphics -
```