Elastic analysis of (concrete) plates on grade

Lesson 2: Numerical implementation and applications

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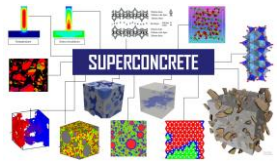

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Ann Arbor, MI - 20 October 2016

Overview

What is the problem we are going to face?

Determining the (generalised) stress fields in concrete slabs on grade (namely, plates on elastic soil) end subjected to sustained and live loads and thermal effects.

How will we approach it?

We will resume the formulation of the Kirchoff model for «thin» elastic plates, introduce the supporting effect of soil and convert the differential equations into their finite difference representation.

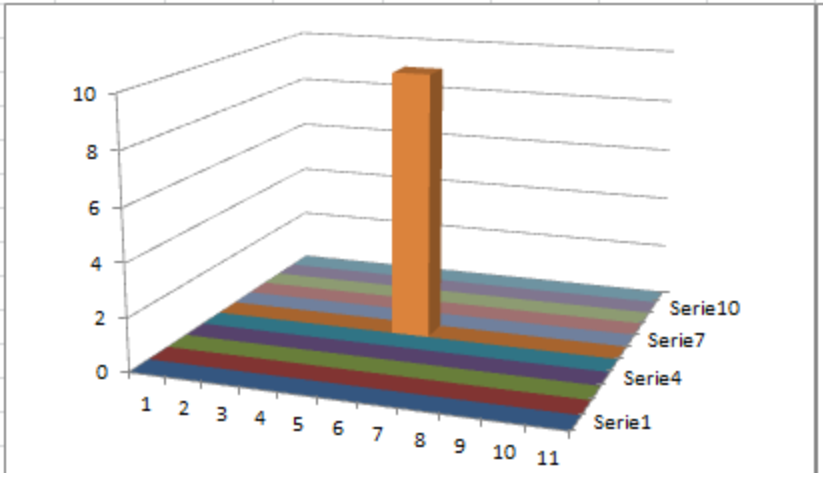
What is the final goal of this presentation?

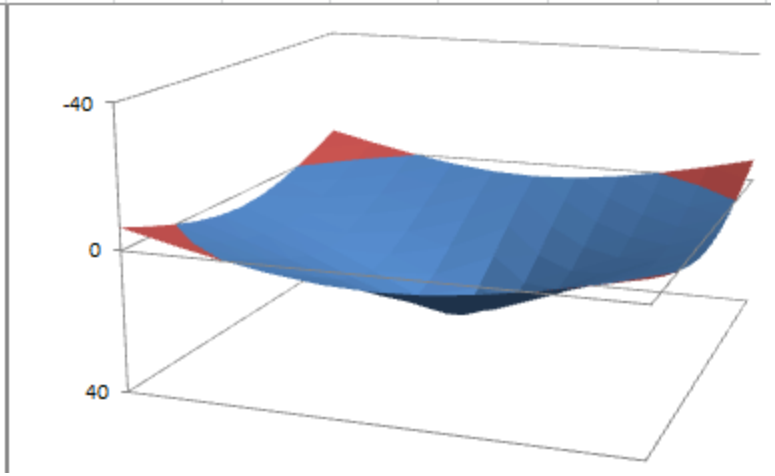
Realising a spreadsheet for analysing concrete slabs on elastic soil, subjected to self-weight, concentrated loads and thermal effects.

Overview

The final goal: a spreadsheet for the analysis of slabs on grade

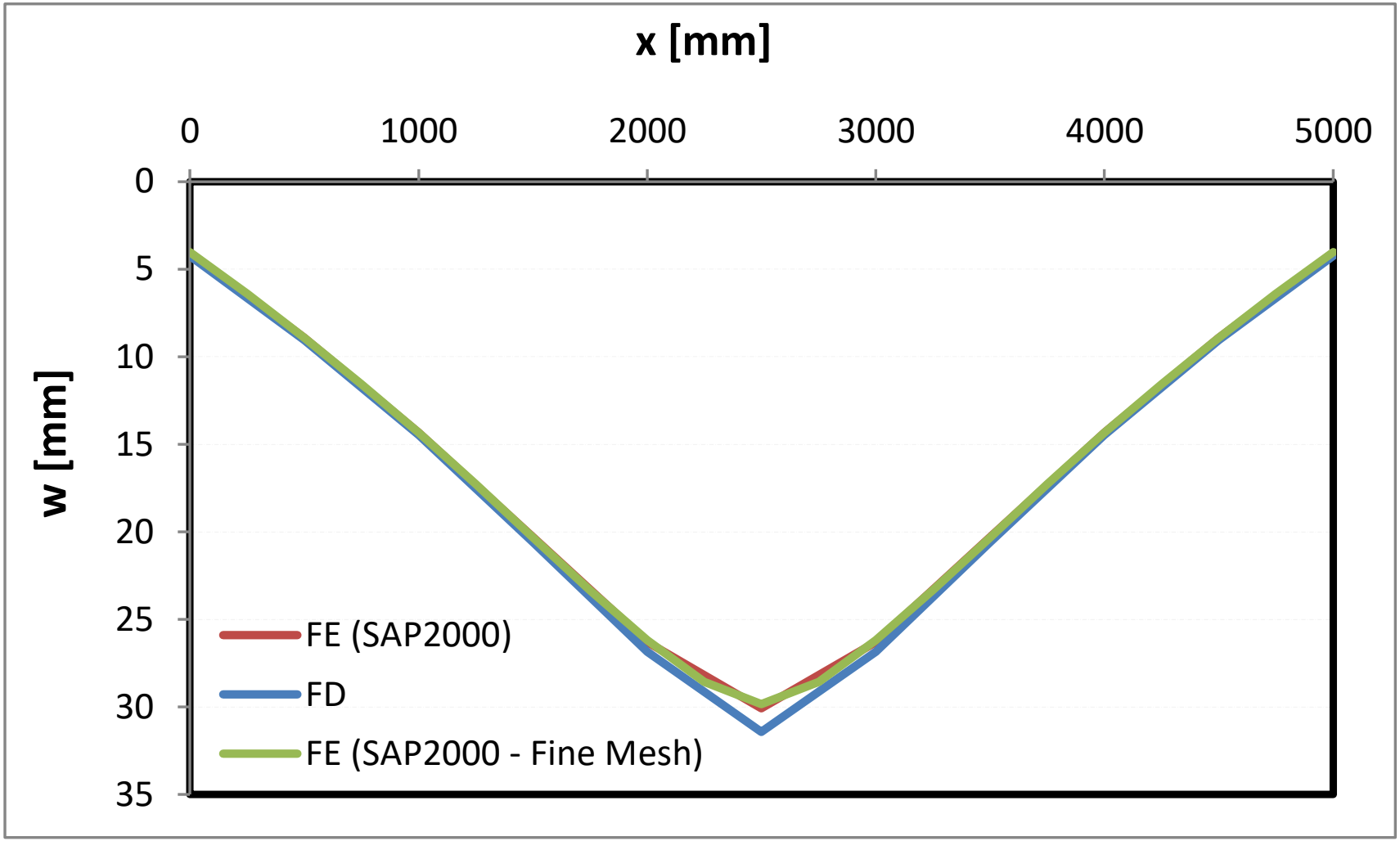
			-1.17E+01	-9.59E+00	-7.48E+00	-6.15E+00	-5.53E+00	-5.42E+00	-5.53E+00	-6.15E+00	-7.48E+00	-9.59E+00	-1.17E+01		
		-1.34E+01	-9.63E+00	-6.60E+00	-4.05E+00	-2.07E+00	-7.89E-01	-3.44E-01	-7.89E-01	-2.07E+00	-4.05E+00	-6.60E+00	-9.63E+00	-1.34E+01	
0	-1.17E+01	-9.63E+00	-6.43E+00	-3.24E+00	-3.56E-01	2.02E+00	3.65E+00	4.23E+00	3.65E+00	2.02E+00	-3.56E-01	-3.24E+00	-6.43E+00	-9.63E+00	-1.17E+01
500	-9.59E+00	-6.60E+00	-3.24E+00	1.83E-01	3.44E+00	6.27E+00	8.29E+00	9.04E+00	8.29E+00	6.27E+00	3.44E+00	1.83E-01	-3.24E+00	-6.60E+00	-9.59E+00
1000	-7.48E+00	-4.05E+00	-3.56E-01	3.44E+00	7.22E+00	1.07E+01	1.34E+01	1.45E+01	1.34E+01	1.07E+01	7.22E+00	3.44E+00	-3.56E-01	-4.05E+00	-7.48E+00
1500	-6.15E+00	-2.07E+00	2.02E+00	6.27E+00	1.07E+01	1.52E+01	1.89E+01	2.06E+01	1.89E+01	1.52E+01	1.07E+01	6.27E+00	2.02E+00	-2.07E+00	-6.15E+00
2000	-5.53E+00	-7.89E-01	3.65E+00	8.29E+00	1.34E+01	1.89E+01	2.40E+01	2.69E+01	2.40E+01	1.89E+01	1.34E+01	8.29E+00	3.65E+00	-7.89E-01	-5.53E+00
2500	-5.42E+00	-3.44E-01	4.23E+00	9.04E+00	1.45E+01	2.06E+01	2.69E+01	3.14E+01	2.69E+01	2.06E+01	1.45E+01	9.04E+00	4.23E+00	-3.44E-01	-5.42E+00
3000	-5.53E+00	-7.89E-01	3.65E+00	8.29E+00	1.34E+01	1.89E+01	2.40E+01	2.69E+01	2.40E+01	1.89E+01	1.34E+01	8.29E+00	3.65E+00	-7.89E-01	-5.53E+00
3500	-6.15E+00	-2.07E+00	2.02E+00	6.27E+00	1.07E+01	1.52E+01	1.89E+01	2.06E+01	1.89E+01	1.52E+01	1.07E+01	6.27E+00	2.02E+00	-2.07E+00	-6.15E+00
4000	-7.48E+00	-4.05E+00	-3.56E-01	3.44E+00	7.22E+00	1.07E+01	1.34E+01	1.45E+01	1.34E+01	1.07E+01	7.22E+00	3.44E+00	-3.56E-01	-4.05E+00	-7.48E+00
4500	-9.59E+00	-6.60E+00	-3.24E+00	1.83E-01	3.44E+00	6.27E+00	8.29E+00	9.04E+00	8.29E+00	6.27E+00	3.44E+00	1.83E-01	-3.24E+00	-6.60E+00	-9.59E+00
5000	-1.17E+01	-9.63E+00	-6.43E+00	-3.24E+00	-3.56E-01	2.02E+00	3.65E+00	4.23E+00	3.65E+00	2.02E+00	-3.56E-01	-3.24E+00	-6.43E+00	-9.63E+00	-1.17E+01
		-1.34E+01	-9.63E+00	-6.60E+00	-4.05E+00	-2.07E+00	-7.89E-01	-3.44E-01	-7.89E-01	-2.07E+00	-4.05E+00	-6.60E+00	-9.63E+00	-1.34E+01	
			-1.17E+01	-9.59E+00	-7.48E+00	-6.15E+00	-5.53E+00	-5.42E+00	-5.53E+00	-6.15E+00	-7.48E+00	-9.59E+00	-1.17E+01		





Overview

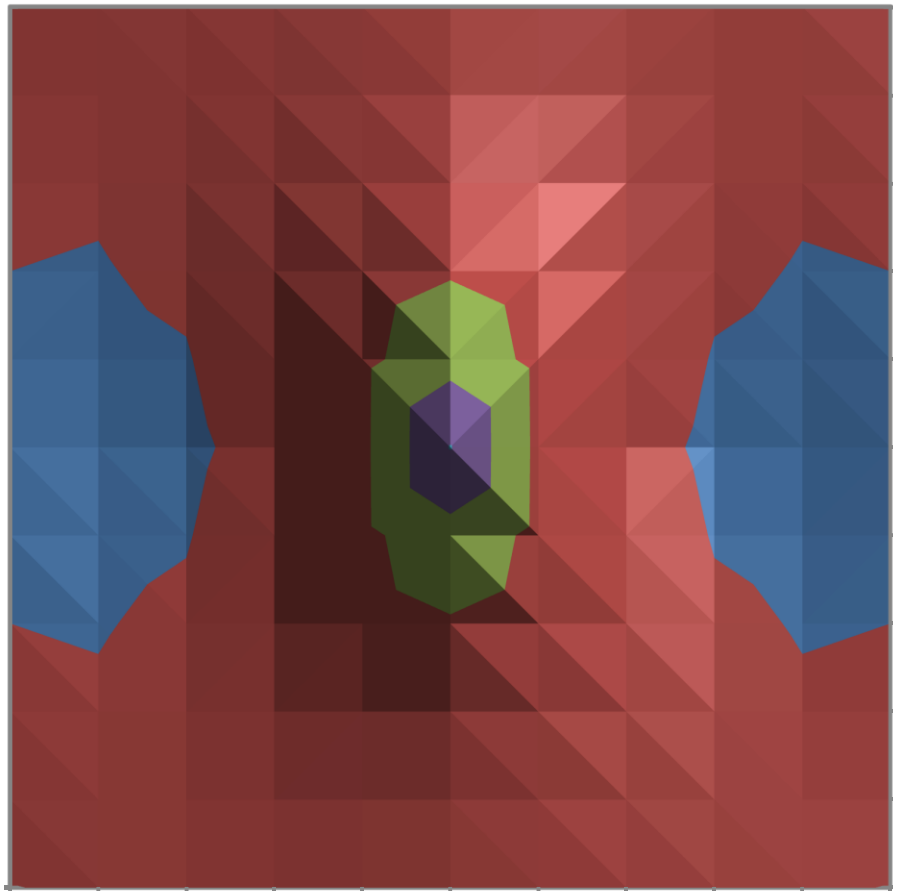
Comparisons with «black-box» commercial FE solutions/1



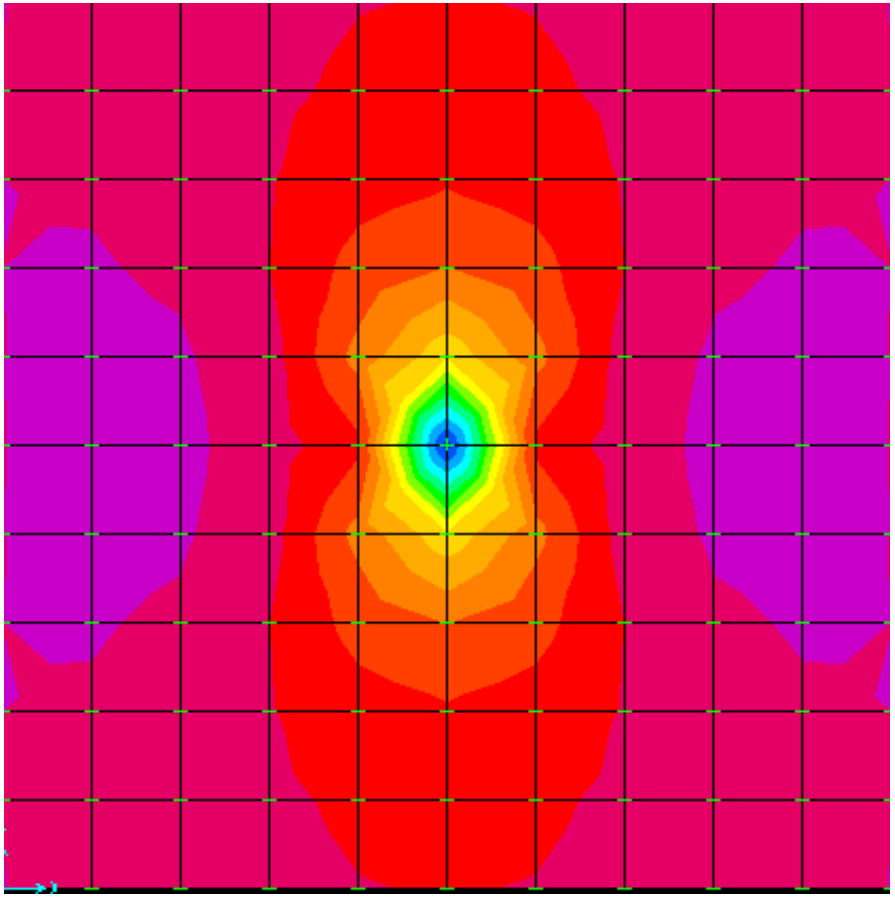
Overview

Comparisons with «black-box» commercial FE solutions/2

Finite Differences – M11



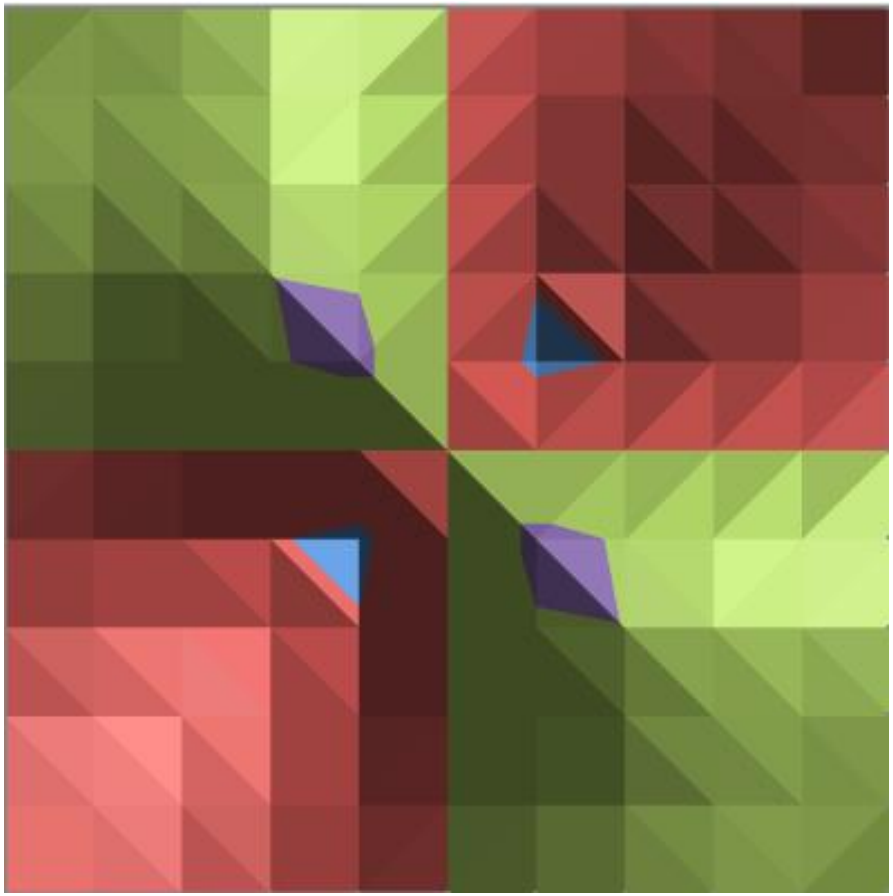
Finite Elements (SAP2000) – M11



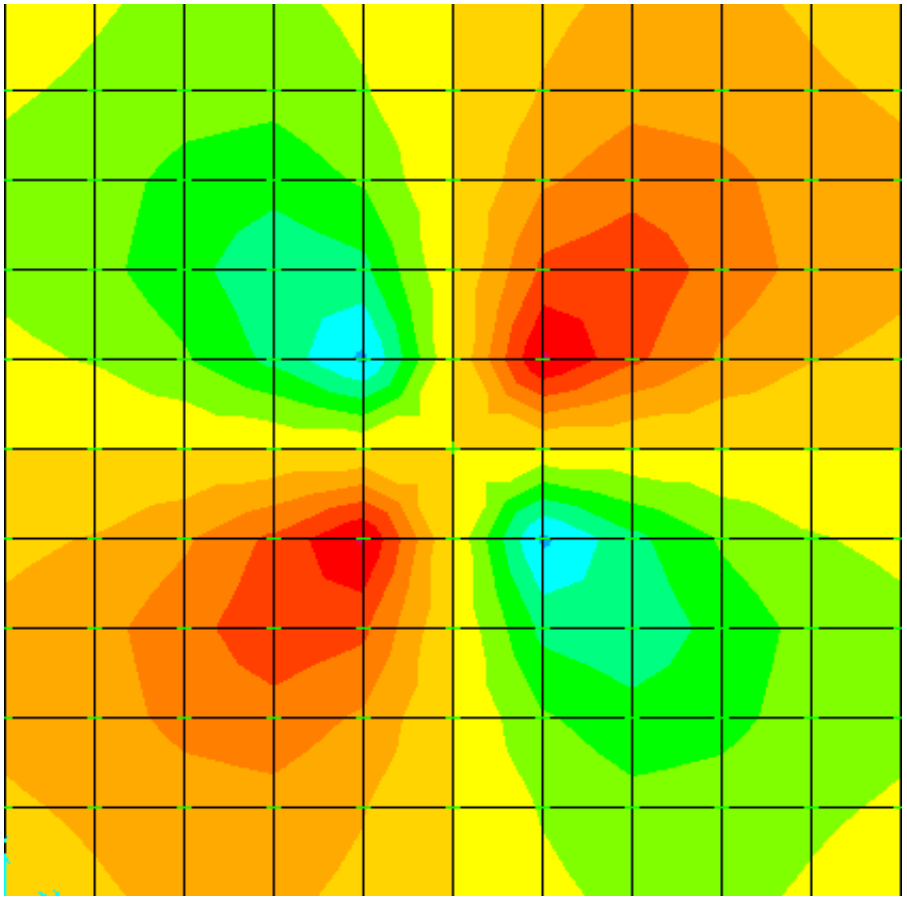
Overview

Comparisons with «black-box» commercial FE solutions/3

Differenze Finite – M12



Elementi Finiti (SAP2000) – M12



Overview

1. Thursday 20 October:
- Summary about the elastic theory of (thin) plates;
- Plates on grade (Winkler soil);
- Summary about Finite Difference (FD) schemes;
- Introduction to the FD solution of elastic plates on grade.

2. Tuesday 25 October:
- Clarifications about Lecture 1;
- Addition of thermal effects;
- Implementation in MS Excel;
- Examples and comparisons.

Overview

- ✓ Clarifications about lecture 1;
- ✓ Addition of thermal effects;
- ✓ Implementation in MS Excel;
- ✓ Examples and comparisons.

Overview

- ✓ Summary about lecture 1;
- ✓ Addition of thermal effects;
- ✓ Implementation in MS Excel;
- ✓ Examples and comparisons.

Summary about Lesson #1

Kirchhoff–Love plate theory: definitions

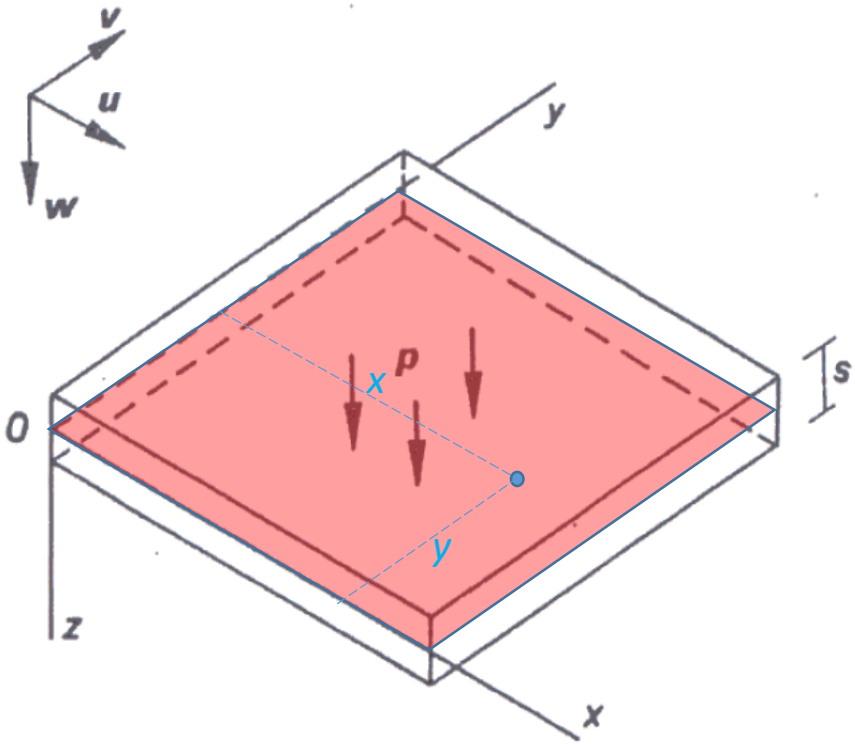
Plates are defined as structural elements with a "small" thickness compared to the planar dimensions.

Thickness-to-width ratio of plates is less than 0.1.

Plate theories are based on this "disparity" in lengths, which allows reducing the full 3D continuum mechanics problem into a 2D problem.

Plate theories aim is to calculate the displacement and stresses fields induced by external actions.

The Kirchhoff–Love theory for "plates" is the equivalent of Euler–Bernoulli for "beams".



*It is assumed that a **mid-surface plane** can be used to represent the 3D into 2D.*

$$w = w(x, y)$$

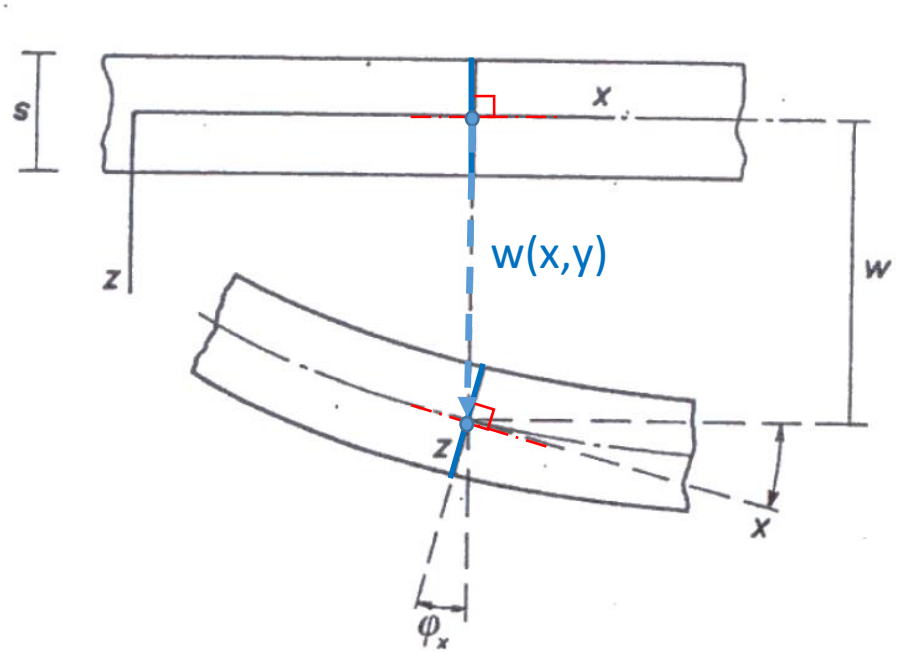
Summary about Lesson #1

Kirchhoff–Love plate theory: assumptions

The following kinematic <u>assumptions</u> that are made in this theory:

- ✓ *straight lines normal to the mid-surface ("chords") remain straight after deformation;*
- ✓ *straight lines normal to the mid-surface remain normal to the mid-surface after deformation;*
- ✓ *the thickness of the plate does not change as a results of deformation.*

Undeformed configuration



Deformed configuration

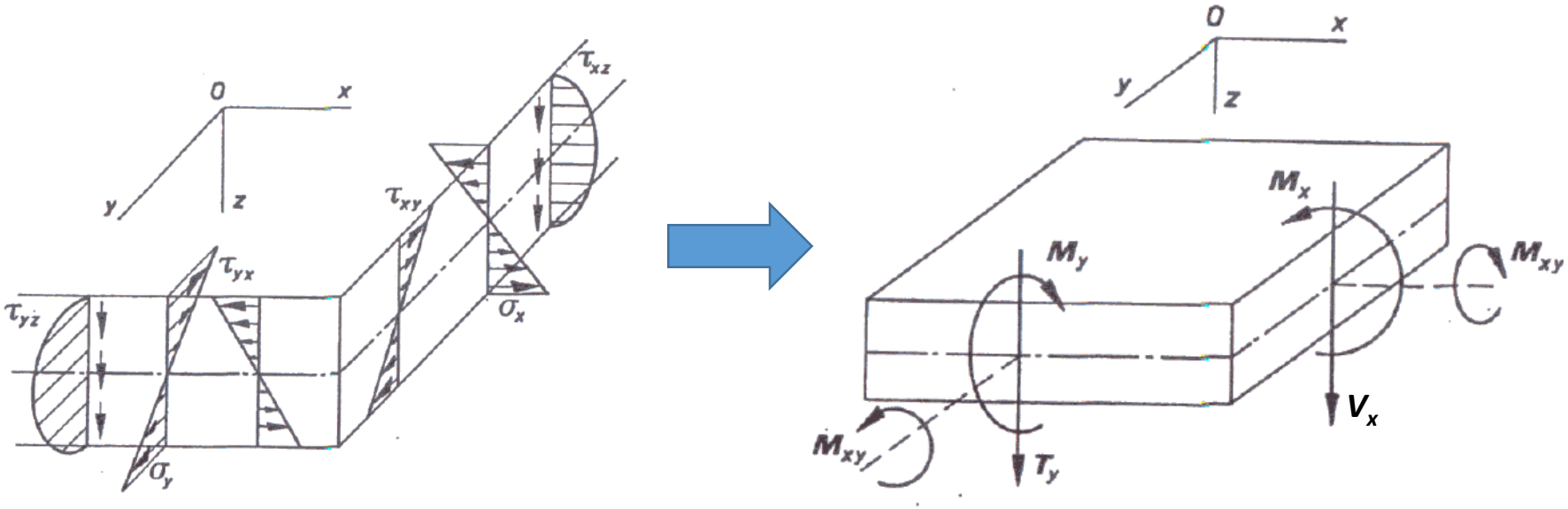
Main Kirchhoff–Love assumption:

$$\varphi_x = -\frac{\partial w}{\partial x}$$

N.B.: φ_x is the rotation of the normal segment in the x-z plane

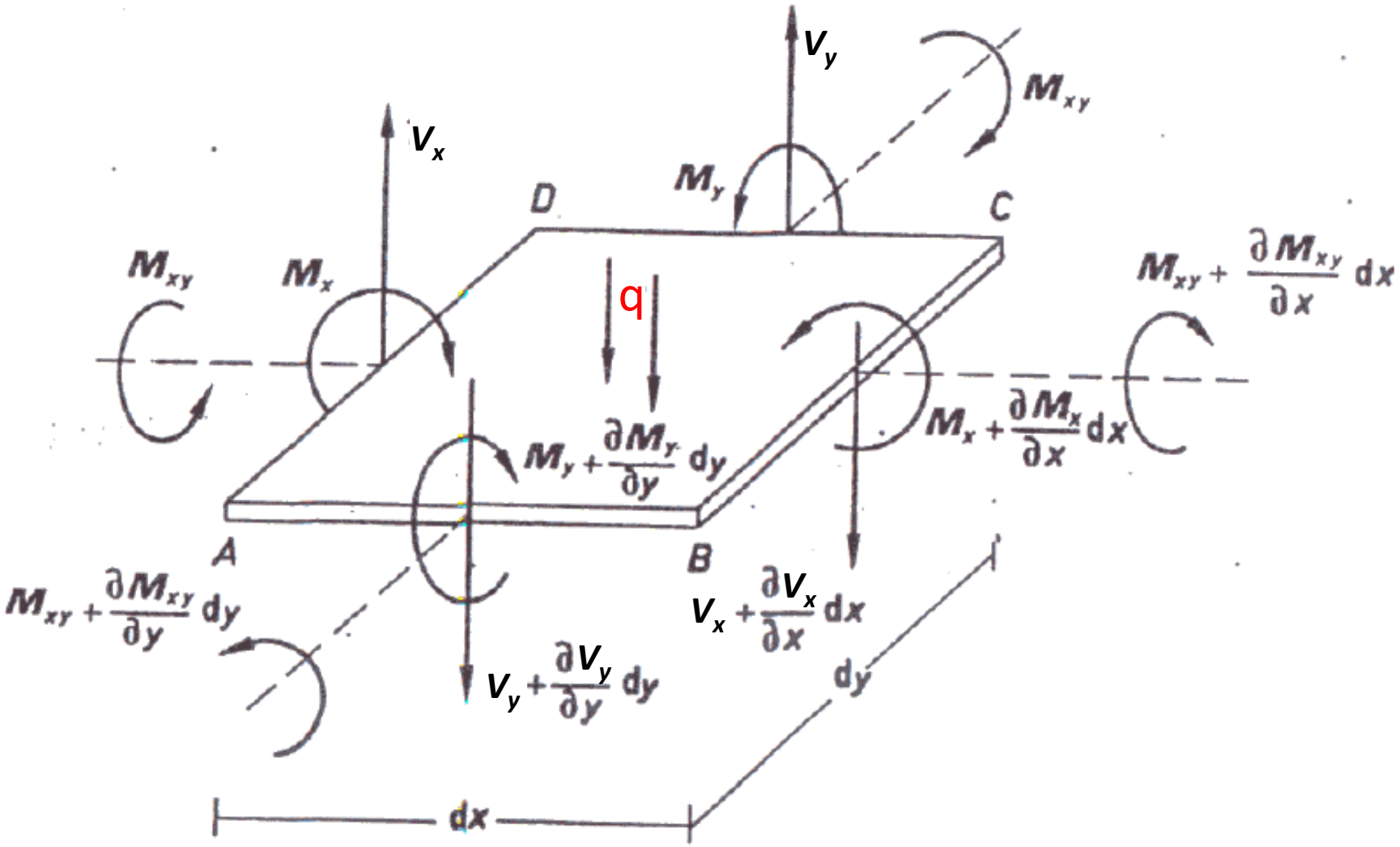
Summary about Lesson #1

Kirchhoff–Love plate theory: generalised stress field/3



Summary about Lesson #1

Kirchhoff–Love plate theory: equilibrium equations/1



Summary about Lesson #1

Kirchhoff–Love plate theory: summary

Beams in bending (Euler-Bernoulli Theory)

***Assumptions**: transverse plane sections remain plane and normal to the longitudinal axis after deformation.*

***Differential equation**:*

$$\frac{d^4 w}{dx^4} = \frac{q}{EI}$$

***Boundary conditions**:*

$$w = 0 \quad \textit{or} \quad V = 0$$

$$\frac{dw}{dx} = 0 \quad \textit{or} \quad M = 0$$

Plates in bending (Kirchhoff-Love Theory)

***Assumptions**: normal straight segments remain straight and normal to the mid-surface after deformation.*

***Differential equation**:*

$$\frac{\partial^4 w}{\partial x^4} + 2 \cdot \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$

***Boundary conditions**:*

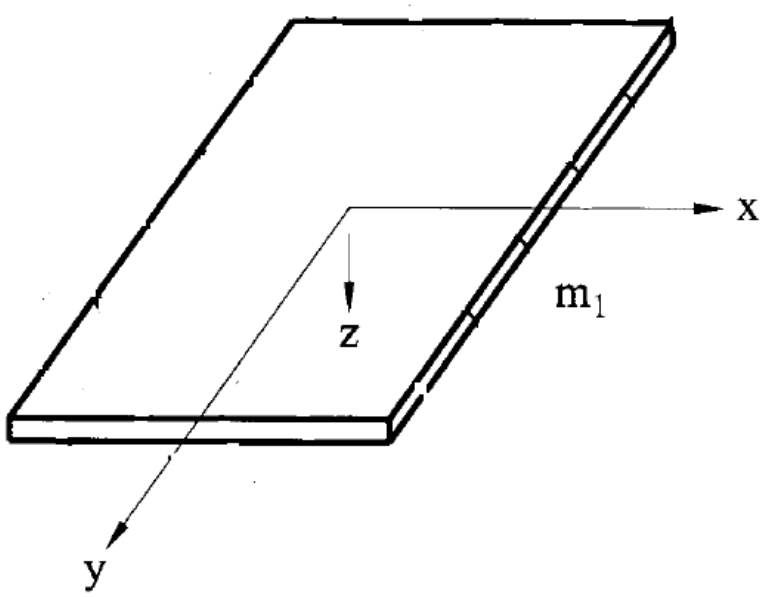
$$w = 0 \quad \textit{or} \quad \begin{matrix} R = 0 \\ M_{xy,edges} = 0 \end{matrix}$$

$$\frac{\partial w}{\partial n} = 0 \quad \textit{or} \quad M_n = 0$$

Summary about Lesson #1

Kirchhoff theory of thin plates

$$\frac{\partial^4 w}{\partial x^4} + 2 \cdot \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$



$q = q(x, y)$ Normal load function

$w = w(x, y)$ Normal displacement function

Equation of plates on an elastic foundation

Winkler soil

$$\sigma = -k_0 \cdot w(x, y)$$

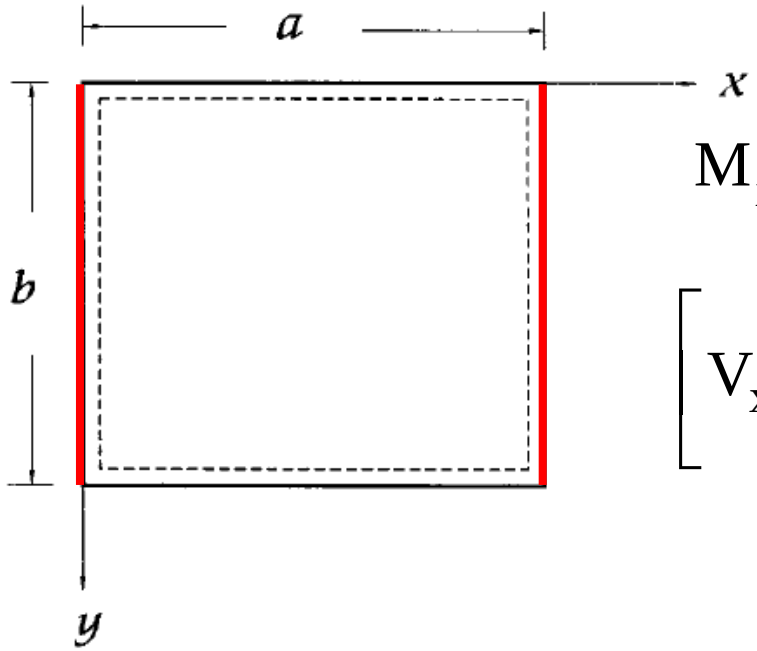
$$\frac{\partial^4 w}{\partial x^4} + 2 \cdot \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{k_0 w}{D} = \frac{q}{D}$$

Summary about Lesson #1

Boundary conditions: free plate

$$M_{x|x=0} = 0$$

$$\left[V_x + \frac{\partial M_{xy}}{\partial y} \right]_{|x=0} = 0$$



$$M_{x|x=a} = 0$$

$$\left[V_x + \frac{\partial M_{xy}}{\partial y} \right]_{|x=a} = 0$$

$$M_x = -D \cdot \left(\frac{\partial^2 w}{\partial x^2} + \nu \cdot \frac{\partial^2 w}{\partial y^2} \right)$$

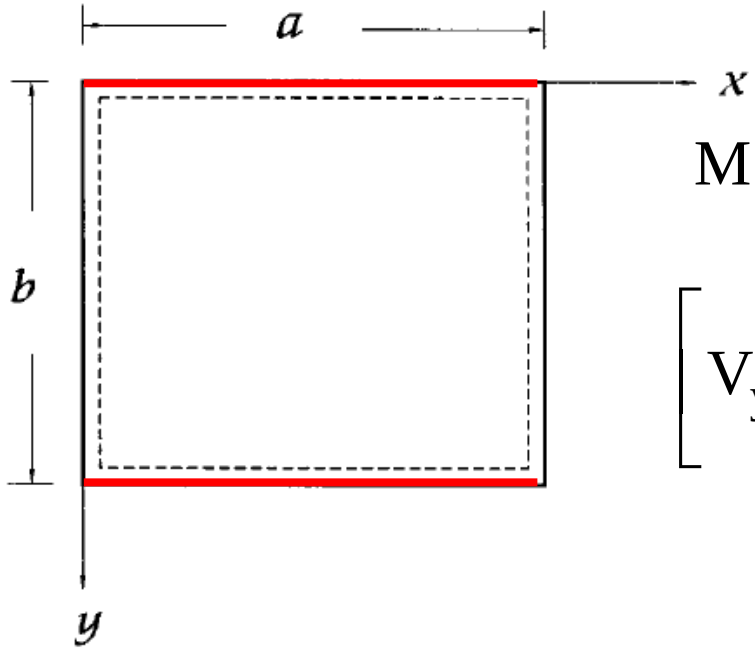
$$R_x = -D \cdot \frac{\partial}{\partial x} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) - D \cdot (1-\nu) \frac{\partial}{\partial y} \frac{\partial^2 w}{\partial x \partial y} = -D \cdot \left(\frac{\partial^3 w}{\partial x^3} + (2-\nu) \cdot \frac{\partial^3 w}{\partial x \partial y^2} \right)$$

Summary about Lesson #1

Boundary conditions: free plate

$$M_{y|y=0} = 0$$

$$\left[V_y + \frac{\partial M_{yx}}{\partial x} \right]_{|y=0} = 0$$



$$M_{y|y=b} = 0$$

$$\left[V_y + \frac{\partial M_{yx}}{\partial x} \right]_{|y=b} = 0$$

$$M_y = -D \cdot \left(\frac{\partial^2 w}{\partial y^2} + \nu \cdot \frac{\partial^2 w}{\partial x^2} \right)$$

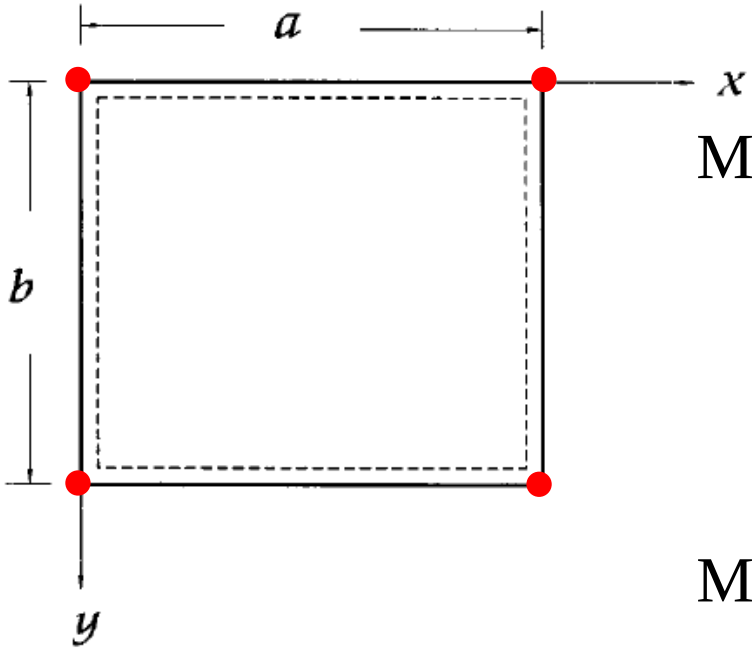
$$R_y = -D \cdot \left(\frac{\partial^3 w}{\partial y^3} + (2 - \nu) \cdot \frac{\partial^3 w}{\partial x^2 \partial y} \right)$$

Summary about Lesson #1

Boundary conditions: free plate

$$M_{xy|x=0,y=0} = 0$$

$$M_{xy|x=a,y=0} = 0$$



$$M_{xy|x=0,y=b} = 0$$

$$M_{xy|x=a,y=b} = 0$$

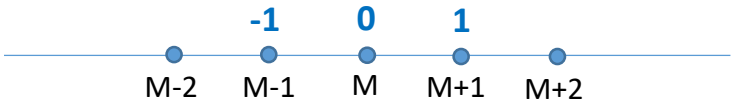
$$M_{xy} = M_{yx} = -(1-\nu)\cdot D\cdot\frac{\partial^2 w}{\partial x \partial y}$$

Summary about Lesson #1

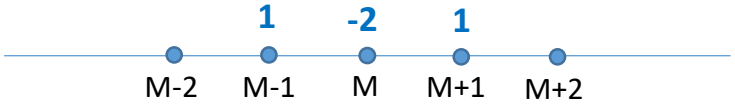
Approximation of higher-order derivatives

$$f_M$$

$$f_M' \approx \frac{f_{M+1} - f_{M-1}}{2 \cdot \Delta x}$$

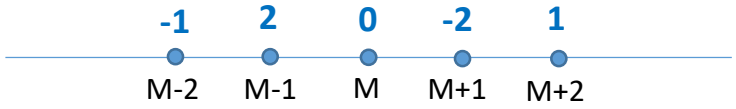


$$f_M'' \approx \frac{f_{M+1} - 2f_M + f_{M-1}}{\Delta x^2}$$

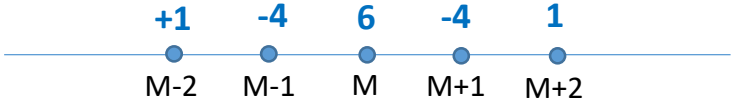


$$f_M''' = (f_M'')' \approx \left(\frac{f_{M+1} - 2f_M + f_{M-1}}{\Delta x^2} \right)' = \frac{1}{\Delta x^2} \cdot \left(\frac{f_{M+2} - f_M}{2 \cdot \Delta x} - 2 \cdot \frac{f_{M+1} - f_{M-1}}{2 \cdot \Delta x} + \frac{f_M - f_{M-2}}{2 \cdot \Delta x} \right)$$

$$f_M''' = \frac{f_{M+2} - 2 \cdot f_{M+1} + 2 \cdot f_{M-1} - f_{M+2}}{2 \cdot \Delta x^3}$$



$$f_M'''' = (f_M'')'' \approx \left(\frac{f_{M+1} - 2f_M + f_{M-1}}{\Delta x^2} \right)'' = \frac{f_{M+2} - 4 \cdot f_{M+1} + 6 \cdot f_M - 4 \cdot f_{M-1} + f_{M-2}}{\Delta x^4}$$



Summary about Lesson #1

Conceptual approach

Differential formulation

Field equation

$$\frac{\partial^4 w}{\partial x^4} + 2 \cdot \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{k_0 w}{D} = \frac{q}{D}$$

Boundary conditions

$$M_x = -D \cdot \left(\frac{\partial^2 w}{\partial x^2} + \nu \cdot \frac{\partial^2 w}{\partial y^2} \right) = 0$$

$$R_x = -D \cdot \left(\frac{\partial^3 w}{\partial x^3} + (2 - \nu) \cdot \frac{\partial^3 w}{\partial x \partial y^2} \right) = 0$$

Main unknown

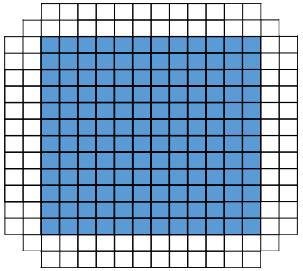
$$w = w(x, y)$$

Finite Difference Discretization

Algebraic conversion

Field equations

Boundary conditions



Main unknowns

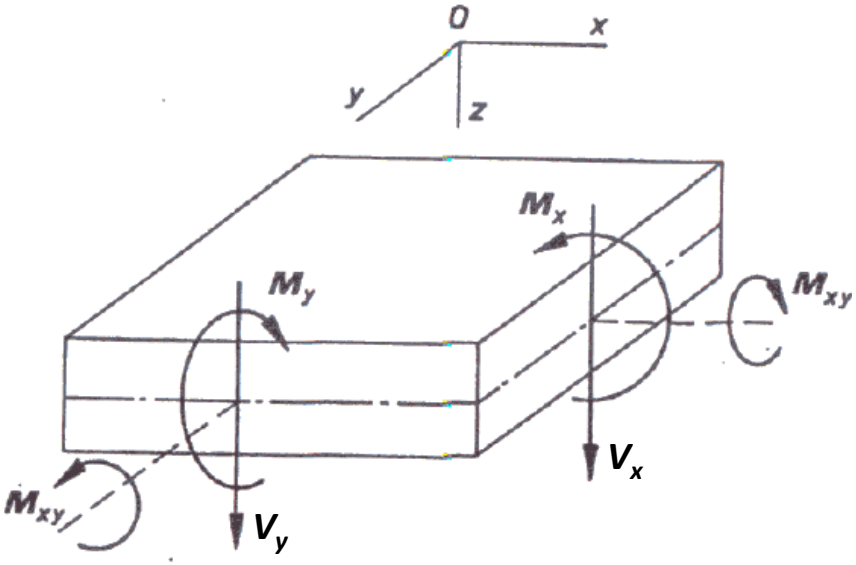
$$w_{M,N}$$

Overview

- ✓ Clarifications about lecture 1;
- ✓ Addition of thermal effects;
- ✓ Implementation in MS Excel;
- ✓ Examples and comparisons.

Thermal effects

Moment-curvature relationships in the absense of thermal effects



In the absence of thermal effects

$$M_x = \int_{-s/2}^{s/2} \sigma_x \cdot z \cdot dz = -D \cdot \left(\frac{\partial^2 w}{\partial x^2} + \nu \cdot \frac{\partial^2 w}{\partial y^2} \right)$$

$$M_y = \int_{-s/2}^{s/2} \sigma_y \cdot z \cdot dz = -D \cdot \left(\frac{\partial^2 w}{\partial y^2} + \nu \cdot \frac{\partial^2 w}{\partial x^2} \right)$$

$$D = \frac{E \cdot s^3}{12 \cdot \left(1 - \nu^2\right)}$$

In a more general sense, in plates, as well as in beams, moments are related to curvatures:

Bending moment in the x-z plane (around the y-axis):

$$M_x = D \cdot \left(\chi_x + \nu \cdot \chi_y \right)$$

Curvature in the y-z plane

$$-\frac{\partial^2 w}{\partial x^2} = \chi_x$$

Bending moment in the y-z plane (around the x-axis):

$$M_y = D \cdot \left(\chi_y + \nu \cdot \chi_x \right)$$

Curvature in the y-z plane

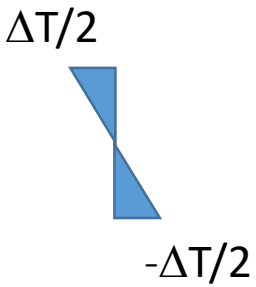
$$-\frac{\partial^2 w}{\partial y^2} = \chi_y$$

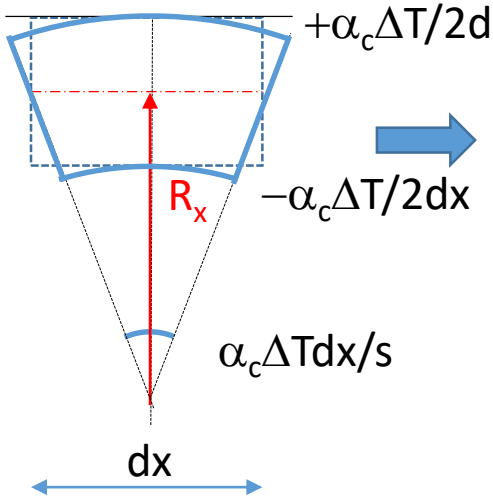
Thermal effects

Deformations due to thermal effects

Deformed shape
(displacements in the x-z plane)

Temperature distribution throughout the slab thickness

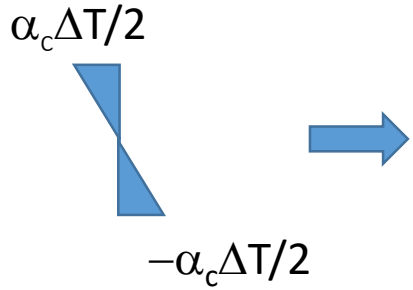




Non-costitutive curvature Induced by thermal effects

$$\chi_{x,th} = \frac{1}{R_x} = -\frac{2}{dx}\cdot\frac{\alpha_c \Delta T}{2}\frac{dx}{s} = -\frac{\alpha_c \Delta T}{s}$$

Distribution on non-costitutive axial strains due to thermal effects



$$\chi_{x,th} = -\frac{\alpha_c \Delta T}{s}$$

α_c coefficient of thermal expnsion

$\alpha_c = 1.2 \cdot 10^{5}\ °C^{-1}$ (Concrete)

Thermal effects

Compatibility conditions

Non-costitutive curvatures Induced by thermal effects

$$\chi_{x,th} = -\alpha_c \frac{\Delta T}{s} \qquad \chi_{y,th} = -\alpha_c \frac{\Delta T}{s}$$

Generalised «compatibility conditions» including thermal effects

$$-\frac{\partial^2 w}{\partial x^2} = \chi_x + \chi_{x,th} \qquad -\frac{\partial^2 w}{\partial y^2} = \chi_y + \chi_{y,th}$$

Total curvature (related to the actual displacements)

«Costitutive» curvature (related to the bending moments)

«Non-costitutive» curvature (induced by thermal effects)

Elastic bending moment-curvature relationships

$$M_x = D \cdot \left(\chi_x + \nu \cdot \chi_y\right) = -D \cdot \left(\frac{\partial^2 w}{\partial x^2} + \nu \cdot \frac{\partial^2 w}{\partial y^2}\right) - D \cdot \left(\chi_{x,th} + \nu \cdot \chi_{y,th}\right)$$

$$M_y = D \cdot \left(\chi_y + \nu \cdot \chi_x\right) = -D \cdot \left(\frac{\partial^2 w}{\partial y^2} + \nu \cdot \frac{\partial^2 w}{\partial x^2}\right) - D \cdot \left(\chi_{y,th} + \nu \cdot \chi_{x,th}\right)$$

Thermal effects

General field equation

$$M_x = D\cdot\left(\chi_x + \nu\cdot\chi_y\right) = -D\cdot\left(\frac{\partial^2 w}{\partial x^2} + \nu\cdot\frac{\partial^2 w}{\partial y^2}\right) - D\cdot\left(\chi_{x,th} + \nu\cdot\chi_{y,th}\right)$$

$$M_y = D\cdot\left(\chi_y + \nu\cdot\chi_x\right) = -D\cdot\left(\frac{\partial^2 w}{\partial y^2} + \nu\cdot\frac{\partial^2 w}{\partial x^2}\right) - D\cdot\left(\chi_{y,th} + \nu\cdot\chi_{x,th}\right)$$

Assumption: temperature is uniform on the plate and it only varies throughout the thickness.
Consequence: the derivatives of $\chi_{x,th}$ and $\chi_{y,th}$ with respect to x and y vanish.

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + q - k_0 w = 0$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - V_x = 0$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - V_y = 0$$

$$\frac{\partial^2 M_x}{\partial x^2} + 2\cdot\frac{\partial^2 M_{xy}}{\partial x\partial y} + \frac{\partial^2 M_y}{\partial y^2} + q - k_0 w = 0$$

$$\frac{\partial^4 w}{\partial x^4} + 2\cdot\frac{\partial^4 w}{\partial x^2\partial y^2} + \frac{\partial^4 w}{\partial y^4} + k_0 w = \frac{q}{D}$$

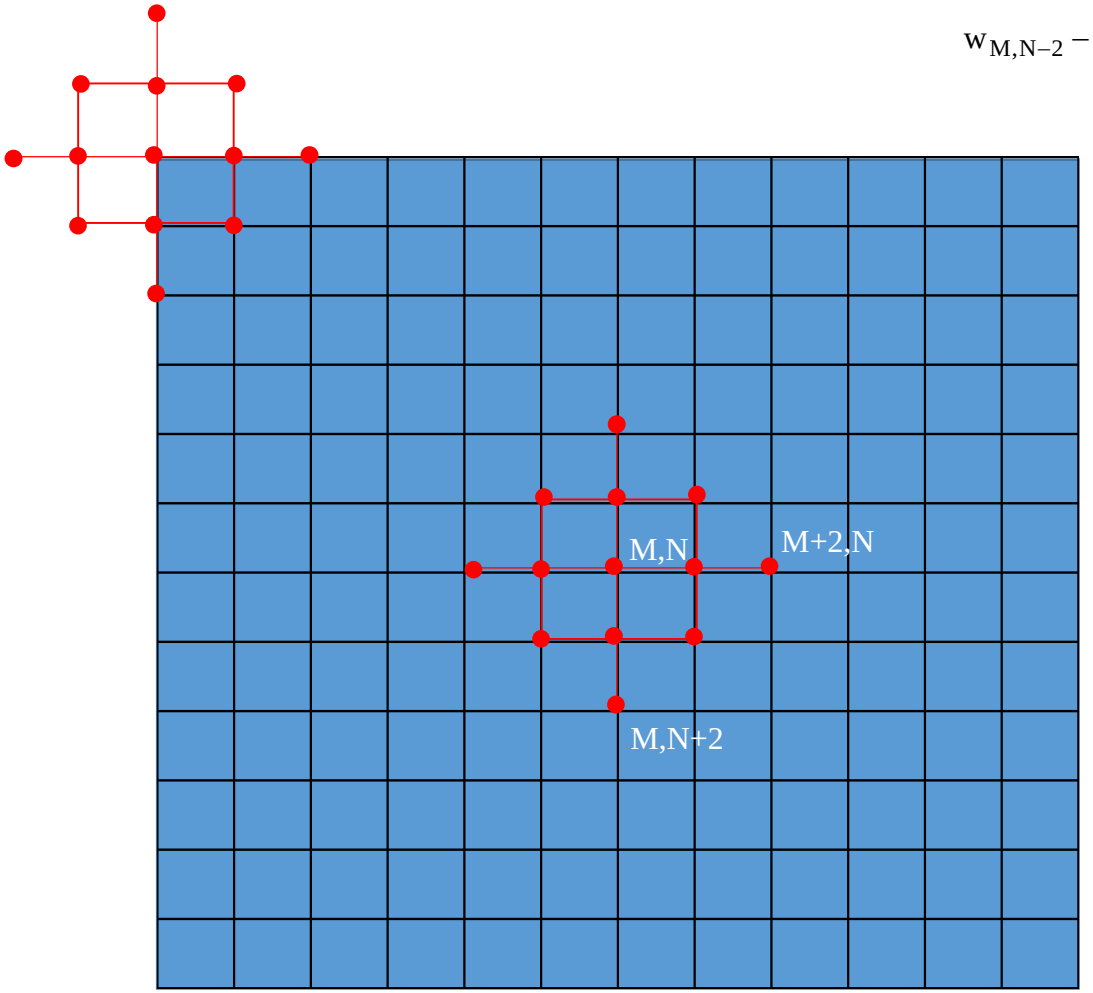
N.B.: no changes occur in the field equation in the case of uniform temperature profile in the x,y plane

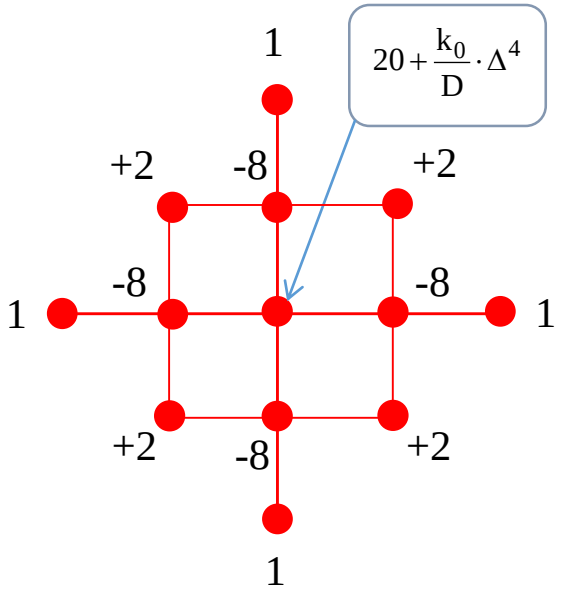
Thermal effescts

Field equation: finite difference discretisation

$$\frac{\partial^4 w}{\partial x^4} + 2\cdot\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{k_0 w}{D} = \frac{q}{D}$$

$$\begin{aligned}
&w_{M+2,N} + \\
&2\cdot w_{M+1,N+1} - 8\cdot w_{M+1,N} + 2\cdot w_{M+1,N-1} + \\
&w_{M,N-2} - 8\cdot w_{M,N-1} + \left(20 + \frac{k_0}{D}\cdot\Delta^4\right)\cdot w_{M,N} - 8\cdot w_{M,N+1} + w_{M,N+2} + \\
&2\cdot w_{M-1,N+1} - 8\cdot w_{M-1,N} + 2\cdot w_{M-1,N-1} + \\
&w_{M-2,N} = \\
&= \frac{q_{M,N}}{D}\cdot\Delta^4
\end{aligned}$$



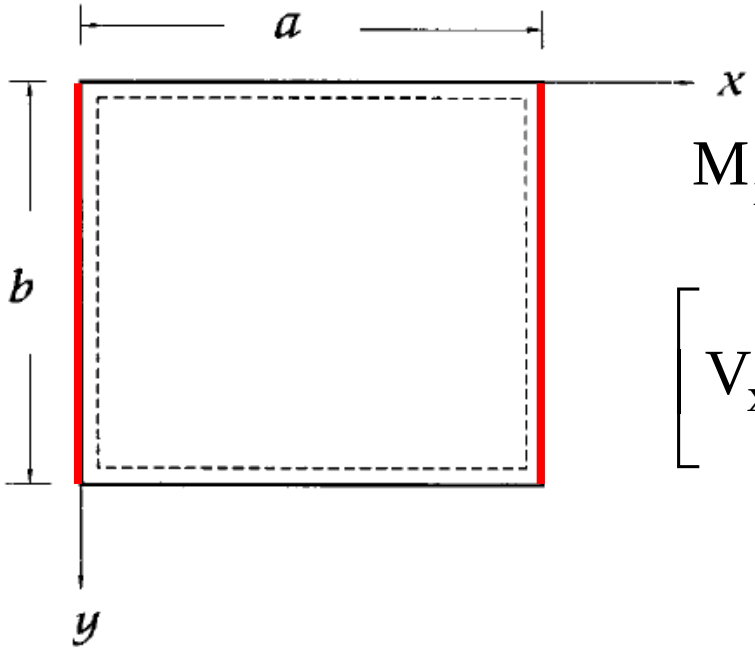


Thermal effects

Boundary conditions: free plate (including thermal effects)

$$M_{x|x=0} = 0 \qquad M_{x|x=a} = 0$$

$$\left[V_x + \frac{\partial M_{xy}}{\partial y} \right]_{|x=0} = 0 \qquad \left[V_x + \frac{\partial M_{xy}}{\partial y} \right]_{|x=a} = 0$$

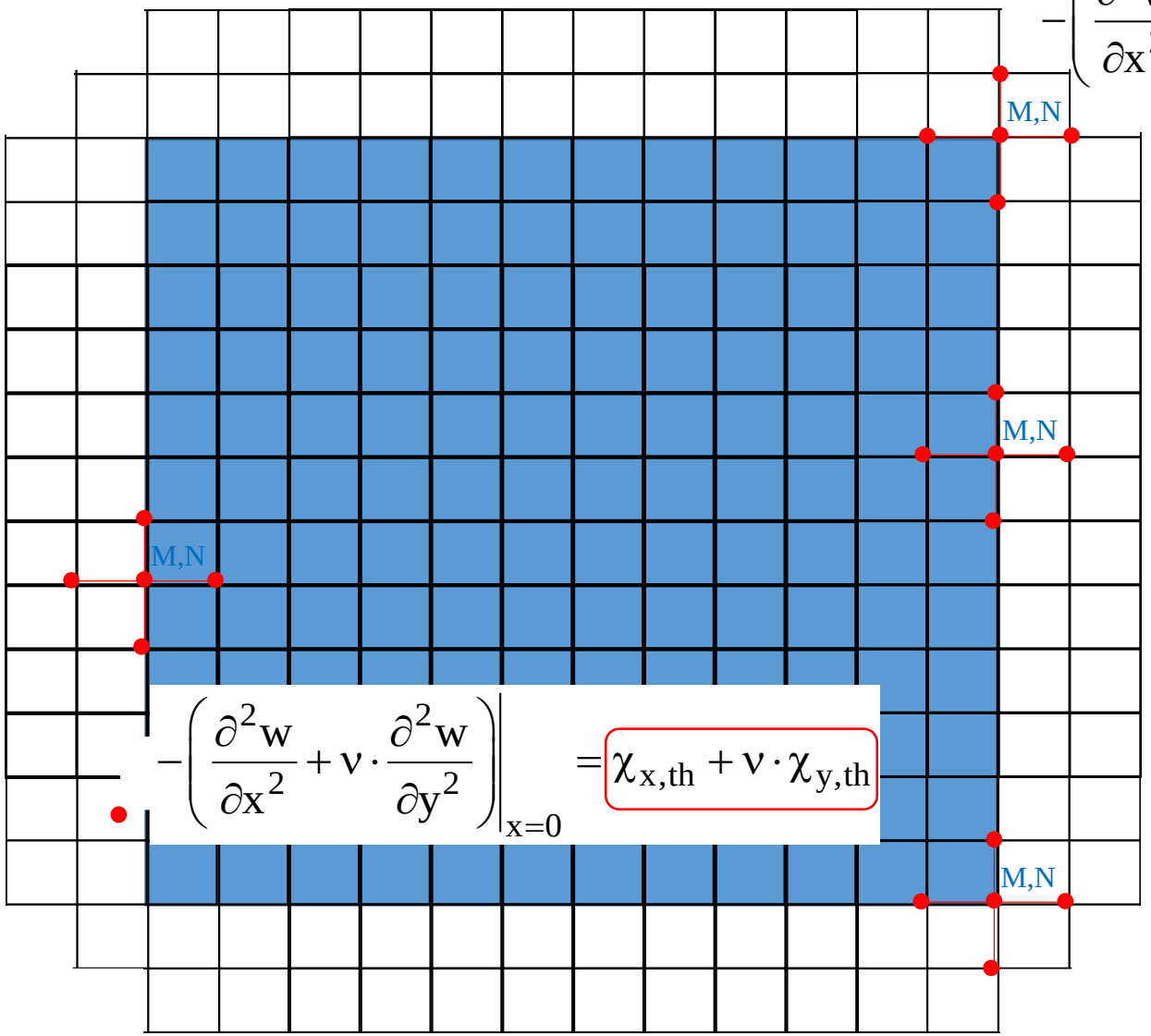


$$M_x = -D \cdot \left(\frac{\partial^2 w}{\partial x^2} + \nu \cdot \frac{\partial^2 w}{\partial y^2} \right) - D \cdot \left(\chi_{x,th} + \nu \cdot \chi_{y,th} \right)$$

$$R_x = -D \cdot \frac{\partial}{\partial x} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) - D \cdot (1-\nu) \frac{\partial}{\partial y} \frac{\partial^2 w}{\partial x \partial y} = -D \cdot \left(\frac{\partial^3 w}{\partial x^3} + (2-\nu) \cdot \frac{\partial^3 w}{\partial x \partial y^2} \right)$$

Thermal effects

Boundary conditions: finite difference discretisation



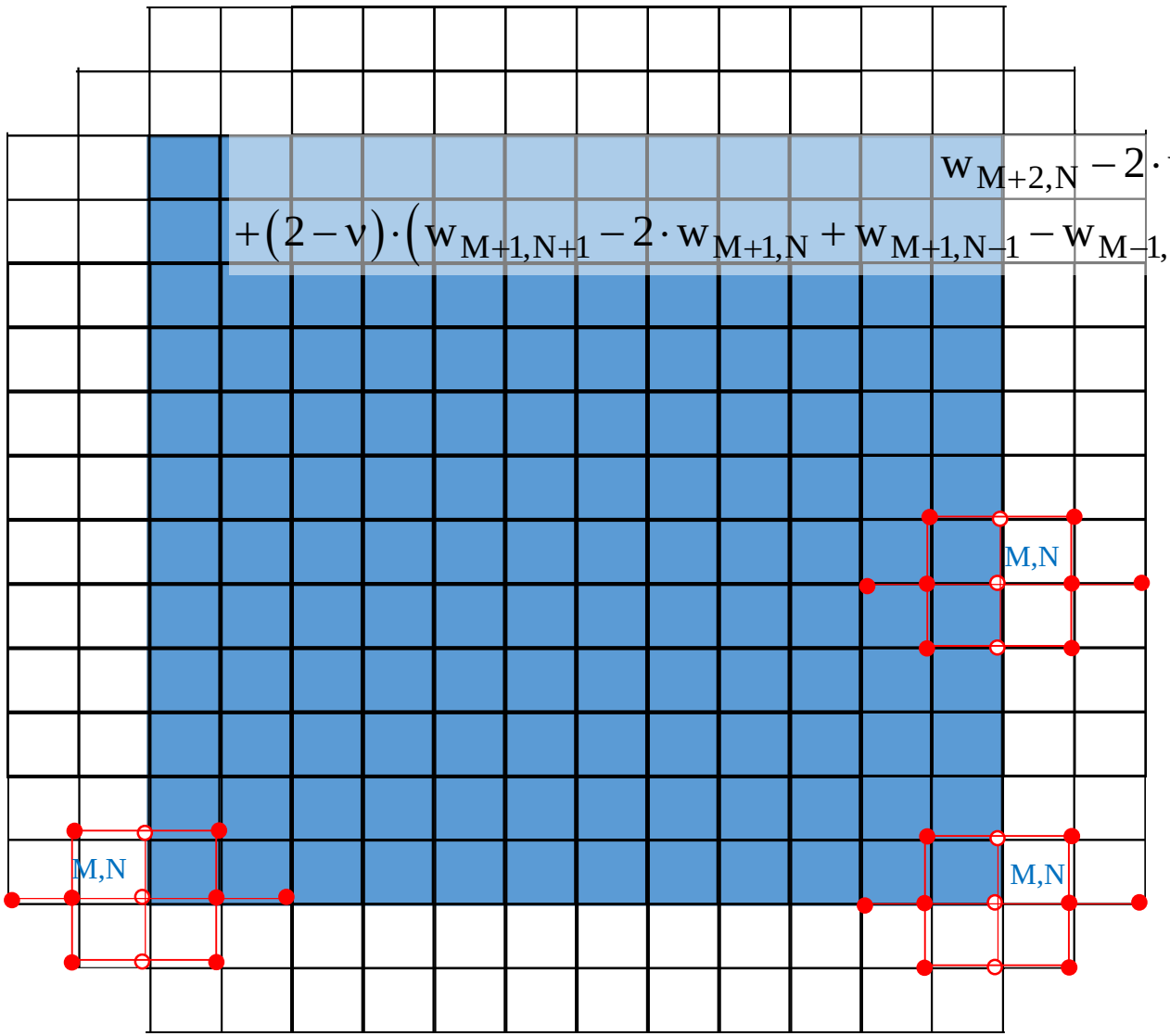
$$-\left(\frac{\partial^2 w}{\partial x^2}+\nu\cdot\frac{\partial^2 w}{\partial y^2}\right)\Bigg|_{x=a}=\chi_{x,th}+\nu\cdot\chi_{y,th}$$

$$\begin{aligned}&\nu\cdot w_{M,N+1}\\ &+w_{M-1,N}-2\cdot(1+\nu)\cdot w_{M,N}+w_{M+1,N}\\ &+\nu\cdot w_{M,N-1}\\ &+\left(\chi_{x,th}+\nu\cdot\chi_{y,th}\right)\cdot\Delta^2=0\end{aligned}$$

$$-\left(\frac{\partial^2 w}{\partial x^2}+\nu\cdot\frac{\partial^2 w}{\partial y^2}\right)\Bigg|_{x=0}=\chi_{x,th}+\nu\cdot\chi_{y,th}$$

Thermal effects

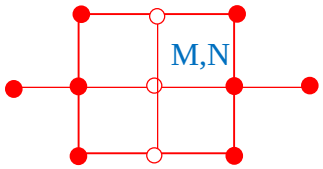
Boundary conditions: finite difference discretisation



$$-D\cdot\left(\frac{\partial^3 w}{\partial x^3}+(2-\nu)\cdot\frac{\partial^3 w}{\partial x\partial y^2}\right)_{x=a}=0$$

$$w_{M+2,N}-2\cdot w_{M+1,N}+2\cdot w_{M-1,N}-w_{M-2,N}+$$
$$+(2-\nu)\cdot\left(w_{M+1,N+1}-2\cdot w_{M+1,N}+w_{M+1,N-1}-w_{M-1,N+1}+2\cdot w_{M-1,N}-w_{M-1,N-1}\right)=0$$

$$-D\cdot\left(\frac{\partial^3 w}{\partial x^3}+(2-\nu)\cdot\frac{\partial^3 w}{\partial x\partial y^2}\right)_{x=0}=0$$



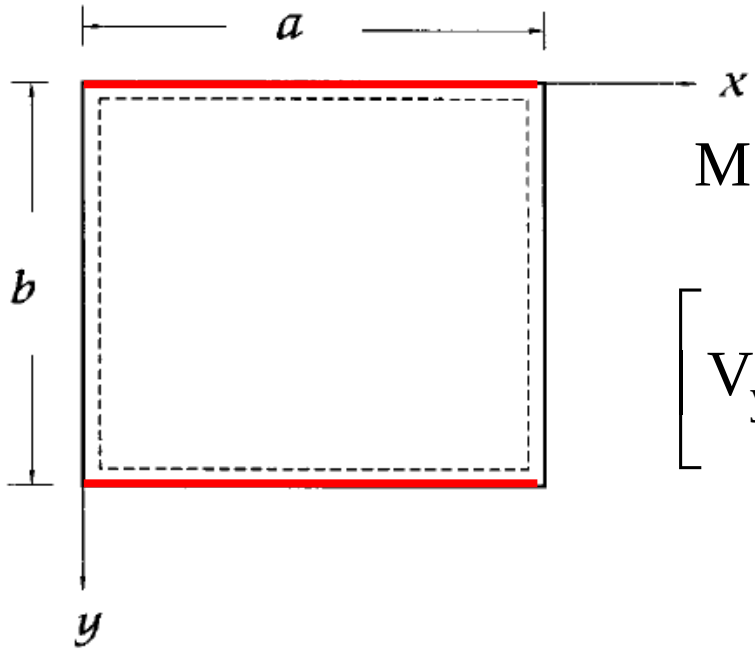
No changes!

Summary about Lesson #1

Boundary conditions: free plate

$$M_{y|y=0} = 0$$

$$\left[V_y + \frac{\partial M_{yx}}{\partial x} \right]_{|y=0} = 0$$



$$M_{y|y=b} = 0$$

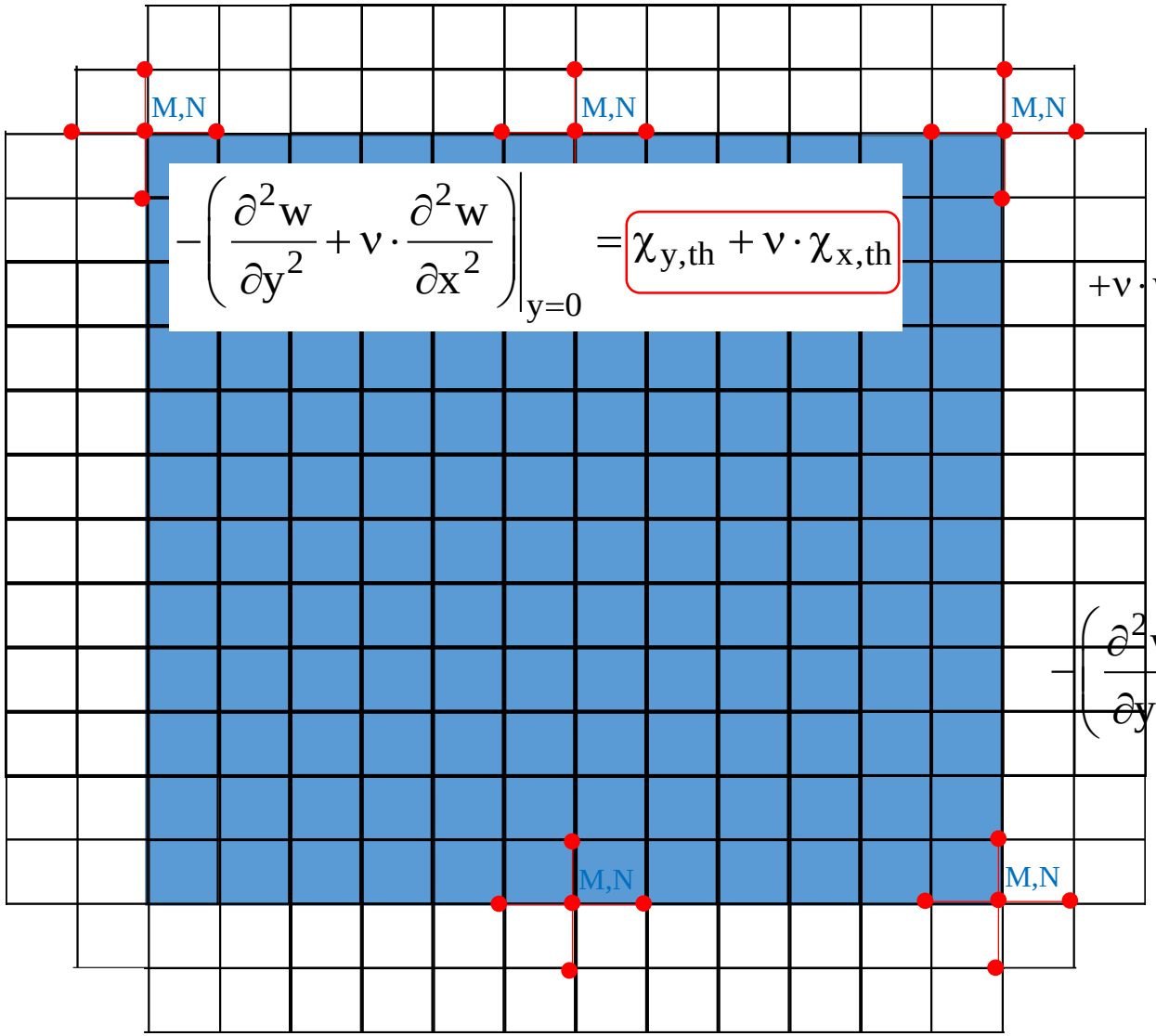
$$\left[V_y + \frac{\partial M_{yx}}{\partial x} \right]_{|y=b} = 0$$

$$M_x = -D \cdot \left(\frac{\partial^2 w}{\partial y^2} + \nu \cdot \frac{\partial^2 w}{\partial x^2} \right) - D \cdot \left(\chi_{y,th} + \nu \cdot \chi_{x,th} \right)$$

$$R_y = -D \cdot \left(\frac{\partial^3 w}{\partial y^3} + (2 - \nu) \cdot \frac{\partial^3 w}{\partial x^2 \partial y} \right)$$

Thermal effects

Boundary conditions: finite difference discretisation



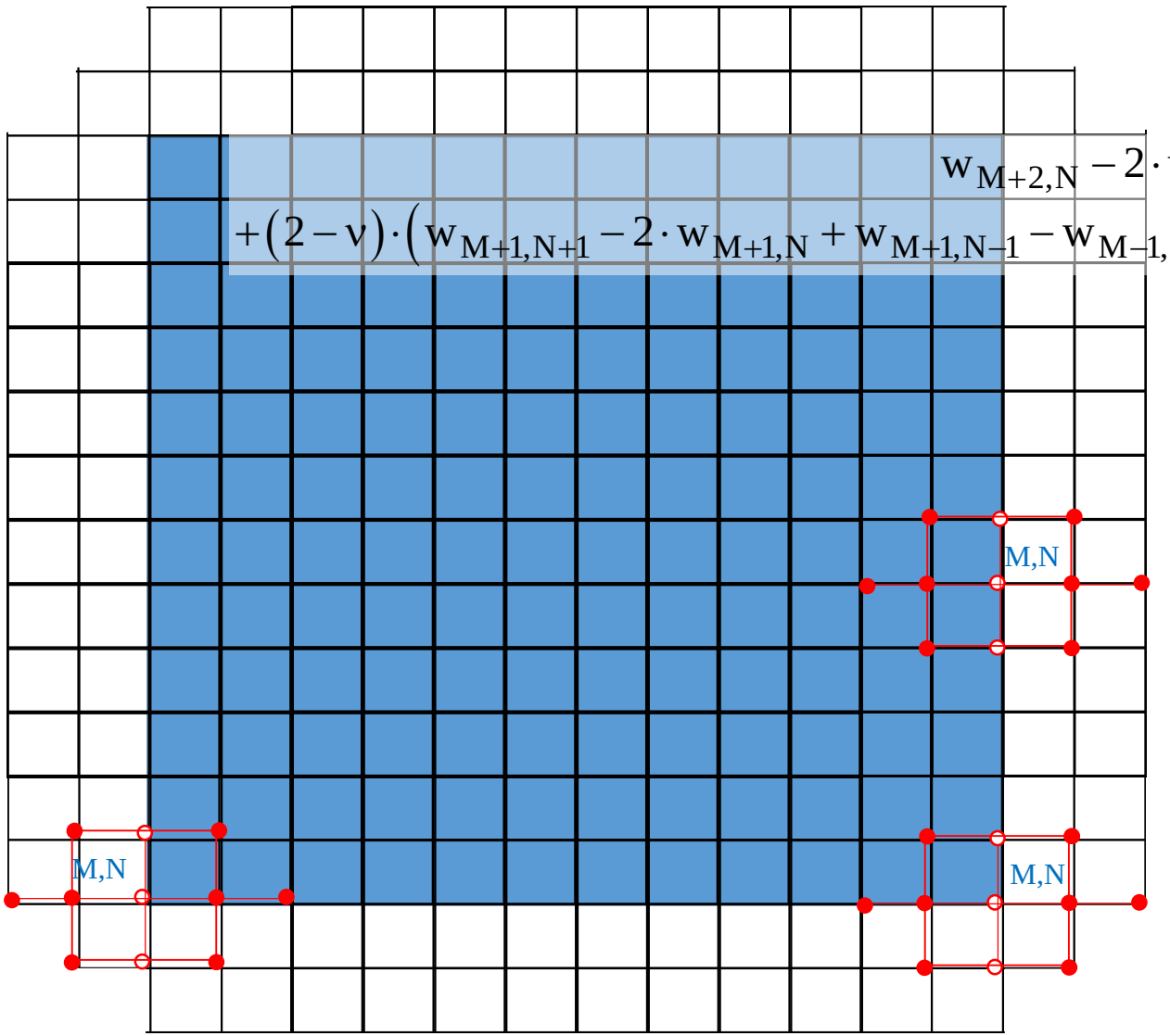
$$-\left(\frac{\partial^2 w}{\partial y^2}+\nu\cdot\frac{\partial^2 w}{\partial x^2}\right)\Bigg|_{y=0}=\chi_{y,th}+\nu\cdot\chi_{x,th}$$

$$w_{M+1,N}+\nu\cdot w_{M,N-1}-2\cdot(1+\nu)\cdot w_{M,N}+\nu\cdot w_{M,N+1}+w_{M-1,N}+\left(\chi_{y,th}+\nu\cdot\chi_{x,th}\right)\cdot\Delta^2=0$$

$$-\left(\frac{\partial^2 w}{\partial y^2}+\nu\cdot\frac{\partial^2 w}{\partial x^2}\right)\Bigg|_{y=b}=\chi_{y,th}+\nu\cdot\chi_{x,th}$$

Thermal effects

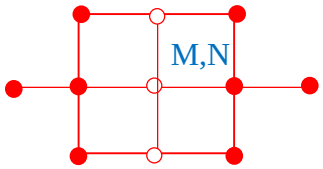
Boundary conditions: finite difference discretisation



$$-D\cdot\left(\frac{\partial^3 w}{\partial x^3}+(2-\nu)\cdot\frac{\partial^3 w}{\partial x\partial y^2}\right)_{x=a}=0$$

$$w_{M+2,N}-2\cdot w_{M+1,N}+2\cdot w_{M-1,N}-w_{M-2,N}+$$
$$+(2-\nu)\cdot\left(w_{M+1,N+1}-2\cdot w_{M+1,N}+w_{M+1,N-1}-w_{M-1,N+1}+2\cdot w_{M-1,N}-w_{M-1,N-1}\right)=0$$

$$-D\cdot\left(\frac{\partial^3 w}{\partial x^3}+(2-\nu)\cdot\frac{\partial^3 w}{\partial x\partial y^2}\right)_{x=0}=0$$



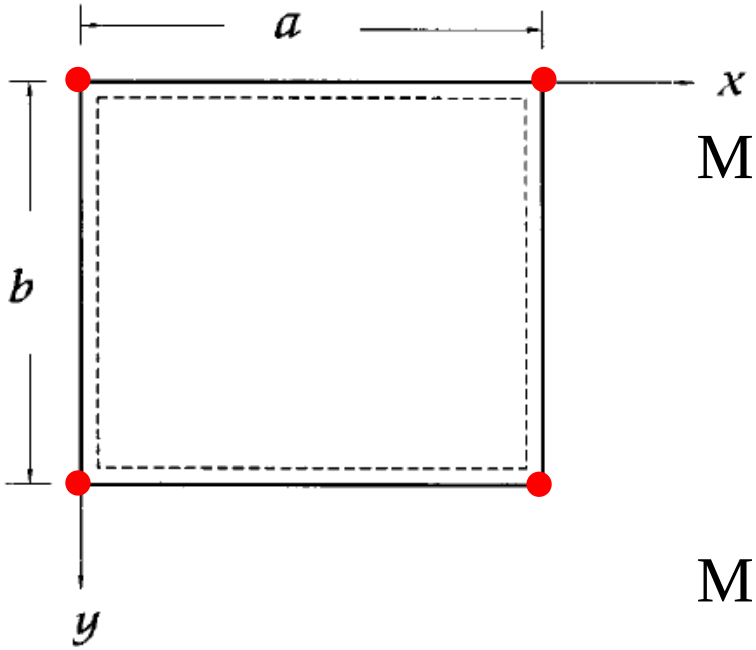
No changes!

Summary about Lesson #1

Boundary conditions: free plate

$$M_{xy|x=0,y=0} = 0$$

$$M_{xy|x=a,y=0} = 0$$



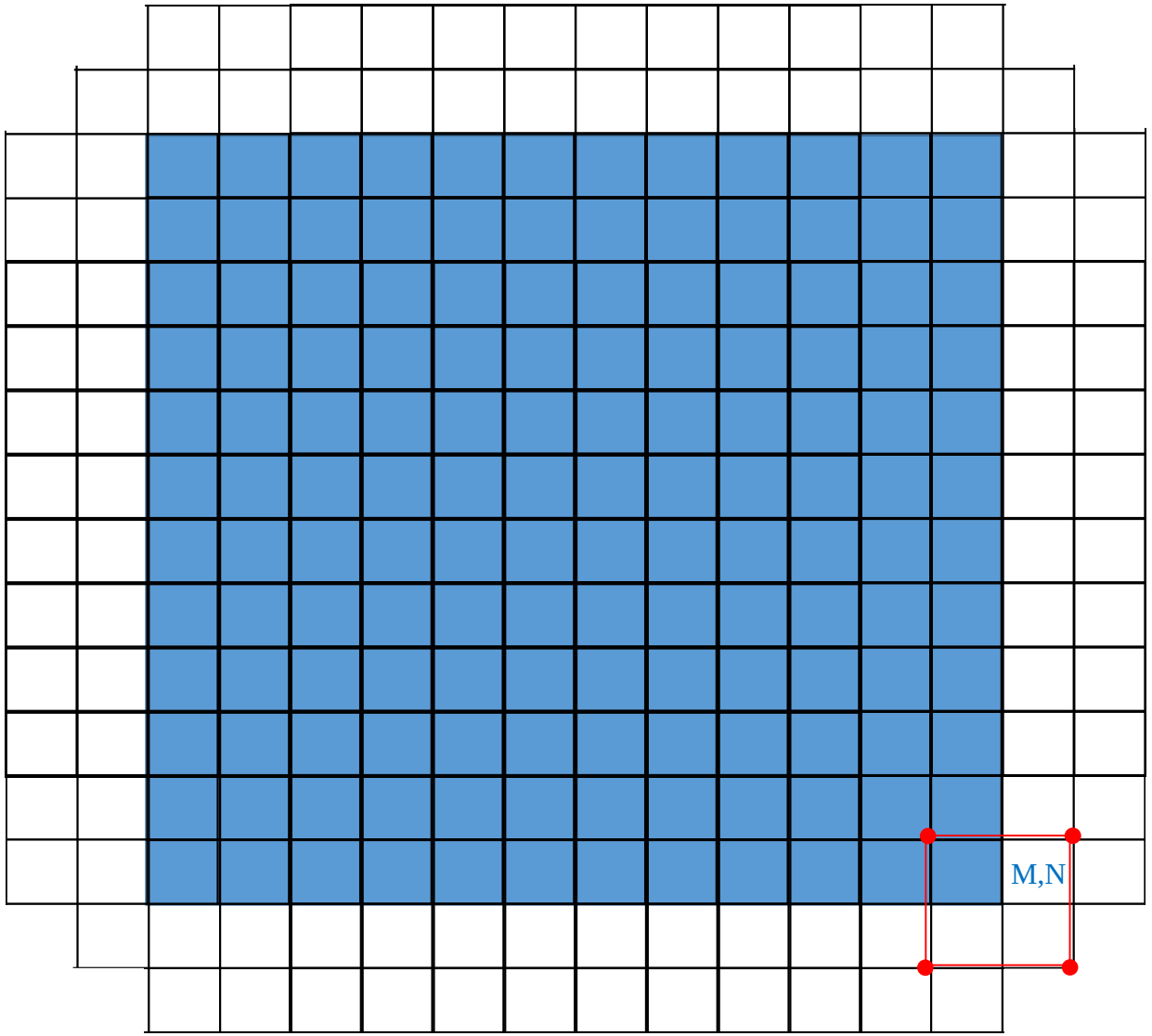
$$M_{xy|x=0,y=b} = 0$$

$$M_{xy|x=a,y=b} = 0$$

$$M_{xy} = M_{yx} = -(1-\nu)\cdot D\cdot\frac{\partial^2 w}{\partial x \partial y}$$

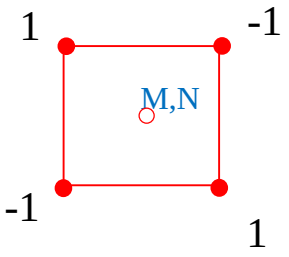
Thermal effects

Boundary conditions: finite difference discretisation



$$-(1-\nu)\cdot D\cdot \left.\frac{\partial^2 w}{\partial x \partial y}\right|_{\substack{x=a \\ y=b}} = 0$$

$$w_{M+1,N+1} - w_{M+1,N-1} + -w_{M-1,N+1} + w_{M-1,N-1} = 0$$



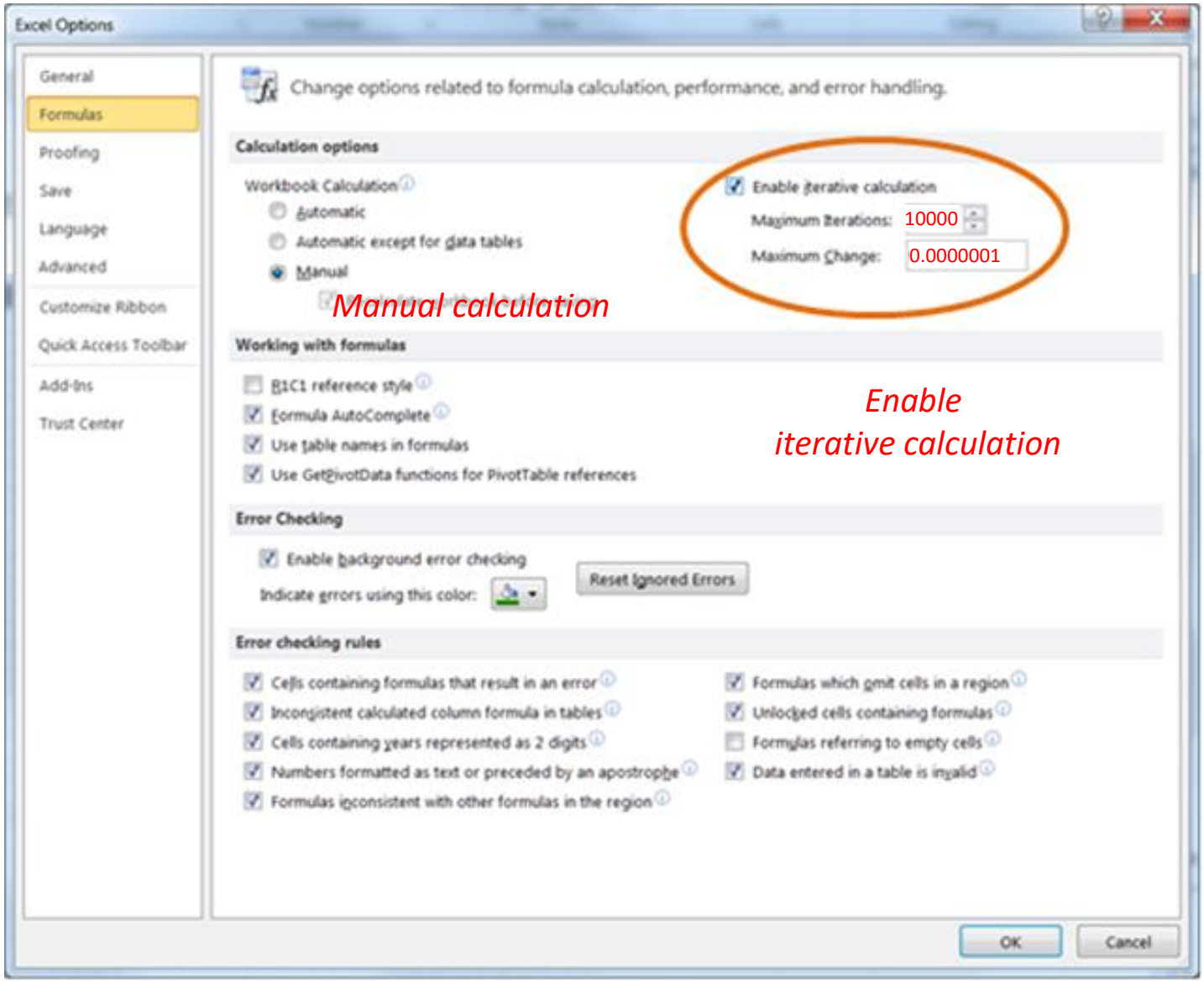
No changes!

Overview

- ✓ Clarifications about lecture 1;
- ✓ Addition of thermal effects;
- ✓ Implementation in MS Excel;
- ✓ Examples and comparisons.

Implementation

Initial setup



Implementation

Input data

Input data:
geometric parameters
and mechanical properties

Loads and actions

Self-weight

Central point force

Thermal effects

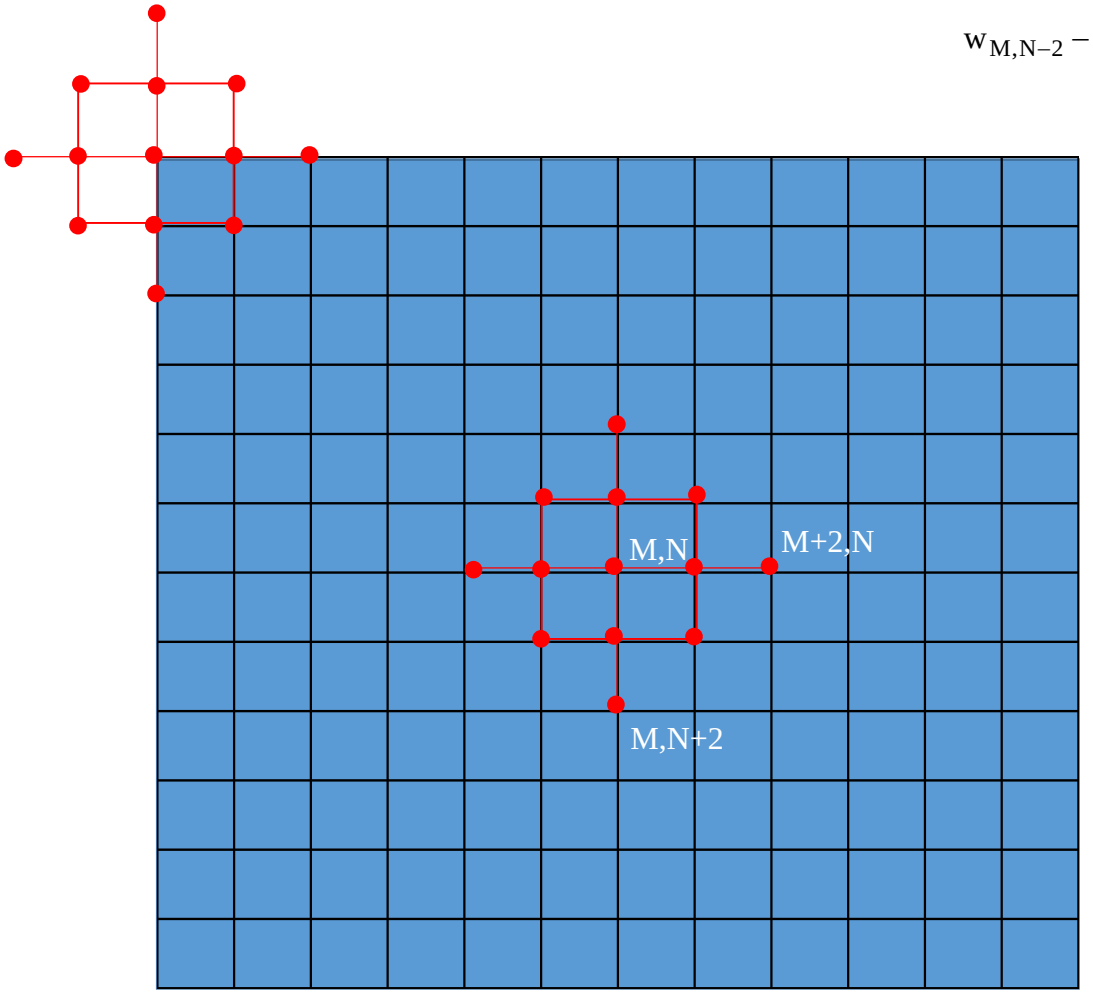
E=	31476	MPa	**Self-weight multiplier**			**Center-point force**			**Thermal effects**		
s=	200	mm	γ_g=	1.00		F=	1000.00	kN	Δt=	25	°C
ν=	0.2		g=	5.00	kN/m				α_c=	1.20E-05	$°C^{-1}$
γ=	25.0	kN/m^3							χ*=	1.50E-06	mm^{-1}
D=	2.186E+10	Nmm									
k_0=	0.01	N/mm^3									
L_x=	5000	mm									
L_y=	5000	mm									
n=	10										
Δ=	500	mm									

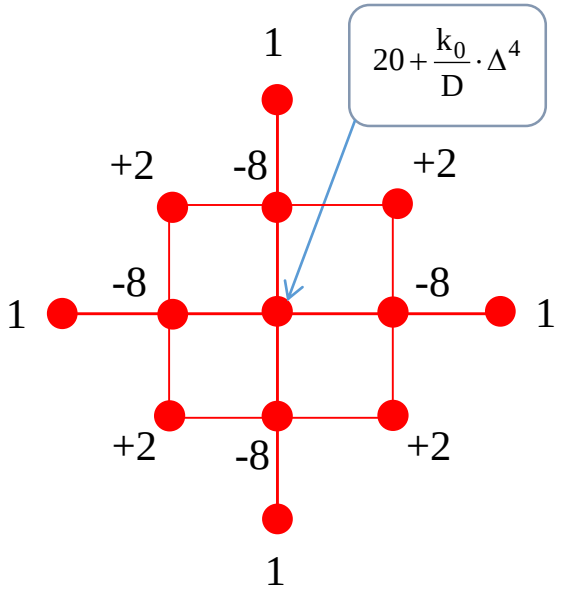
Implementation

Field equation: finite difference discretisation

$$\frac{\partial^4 w}{\partial x^4} + 2\cdot\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{k_0 w}{D} = \frac{q}{D}$$

$$\begin{aligned} & w_{M+2,N} + \\ & 2\cdot w_{M+1,N+1} - 8\cdot w_{M+1,N} + 2\cdot w_{M+1,N-1} + \\ & w_{M,N-2} - 8\cdot w_{M,N-1} + \left(20 + \frac{k_0}{D}\cdot\Delta^4\right)\cdot w_{M,N} - 8\cdot w_{M,N+1} + w_{M,N+2} + \\ & 2\cdot w_{M-1,N+1} - 8\cdot w_{M-1,N} + 2\cdot w_{M-1,N-1} + \\ & w_{M-2,N} = \\ & = \frac{q_{M,N}}{D}\cdot\Delta^4 \end{aligned}$$





Implementation

Field equation: spreadsheet implementation

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1				*0*	*500*	*1000*	*1500*	*2000*	*2500*	*3000*	*3500*	*4000*	*4500*	*5000*		
2				4.55E+00	4.20E+00	4.20E+00	4.26E+00	4.30E+00	4.30E+00	4.30E+00	4.26E+00	4.20E+00	4.20E+00	4.55E+00		
3			3.89E+00	3.47E+00	3.28E+00	3.33E+00	3.50E+00	3.68E+00	3.75E+00	3.68E+00	3.50E+00	3.33E+00	3.28E+00	3.47E+00	3.89E+00	
4	*0*	4.55E+00	3.47E+00	2.93E+00	2.77E+00	2.88E+00	3.13E+00	3.38E+00	3.48E+00	3.38E+00	3.13E+00	2.88E+00	2.77E+00	2.93E+00	3.47E+00	4.55E+00
5	*500*	4.20E+00	3.28E+00	2.77E+00	2.66E+00	2.86E+00	3.22E+00	3.56E+00	3.70E+00	3.56E+00	3.22E+00	2.86E+00	2.66E+00	2.77E+00	3.28E+00	4.20E+00
6	*1000*	4.20E+00	3.33E+00	2.88E+00	2.86E+00	3.18E+00	3.72E+00	4.22E+00	4.45E+00	4.22E+00	3.72E+00	3.18E+00	2.86E+00	2.88E+00	3.33E+00	4.20E+00
7	*1500*	4.26E+00	3.50E+00	3.13E+00	3.22E+00	3.72E+00	4.49E+00	5.27E+00	5.65E+00	5.27E+00	4.49E+00	3.72E+00	3.22E+00	3.13E+00	3.50E+00	4.26E+00
8	*2000*	4.30E+00	3.68E+00	3.38E+00	3.56E+00	4.22E+00	5.27E+00	6.42E+00	7.09E+00	6.42E+00	5.27E+00	4.22E+00	3.56E+00	3.38E+00	3.68E+00	4.30E+00
9	*2500*	=(Load!I9/Data!$B$5*Data!$B$12^4-(-8*I8-8*J9-8*I10-8*H9+2*J8+2*J10+2*H10+2*H8+I7+K9+I11+G9))/(20+Data!$B$6/Data!$B$5*Data!$B$12^4)														
10	*3000*	4.30E+00	3.68E+00	3.38E+00	3.56E+00	4.22E+00	5.27E+00	6.42E+00	7.09E+00	6.42E+00	5.27E+00	4.22E+00	3.56E+00	3.38E+00	3.68E+00	4.30E+00
11	*3500*	4.26E+00	3.50E+00	3.13E+00	3.22E+00	3.72E+00	4.49E+00	5.27E+00	5.65E+00	5.27E+00	4.49E+00	3.72E+00	3.22E+00	3.13E+00	3.50E+00	4.26E+00
12	*4000*	4.20E+00	3.33E+00	2.88E+00	2.86E+00	3.18E+00	3.72E+00	4.22E+00	4.45E+00	4.22E+00	3.72E+00	3.18E+00	2.86E+00	2.88E+00	3.33E+00	4.20E+00
13	*4500*	4.20E+00	3.28E+00	2.77E+00	2.66E+00	2.86E+00	3.22E+00	3.56E+00	3.70E+00	3.56E+00	3.22E+00	2.86E+00	2.66E+00	2.77E+00	3.28E+00	4.20E+00
14	*5000*	4.55E+00	3.47E+00	2.93E+00	2.77E+00	2.88E+00	3.13E+00	3.38E+00	3.48E+00	3.38E+00	3.13E+00	2.88E+00	2.77E+00	2.93E+00	3.47E+00	4.55E+00
15			3.89E+00	3.47E+00	3.28E+00	3.33E+00	3.50E+00	3.68E+00	3.75E+00	3.68E+00	3.50E+00	3.33E+00	3.28E+00	3.47E+00	3.89E+00	
16				4.55E+00	4.20E+00	4.20E+00	4.26E+00	4.30E+00	4.30E+00	4.30E+00	4.26E+00	4.20E+00	4.20E+00	4.55E+00		

3.18E+00	3.72E+00	4.22E+00	4.45E+00	4.22E+00	3.72E+00	3.18E+00
3.72E+00	4.49E+00	5.27E+00	5.65E+00	5.27E+00	4.49E+00	3.72E+00
4.22E+00	5.27E+00	6.42E+00	7.09E+00	6.42E+00	5.27E+00	4.22E+00
4-(-8*I8-8*J9-8*I10-8*H9+2*J8+2*J10+2*H10+2*H8+I7+K9+I11+G9))/(20+Dat						
4.22E+00	5.27E+00	6.42E+00	7.09E+00	6.42E+00	5.27E+00	4.22E+00
3.72E+00	4.49E+00	5.27E+00	5.65E+00	5.27E+00	4.49E+00	3.72E+00
3.18E+00	3.72E+00	4.22E+00	4.45E+00	4.22E+00	3.72E+00	3.18E+00

Implementation

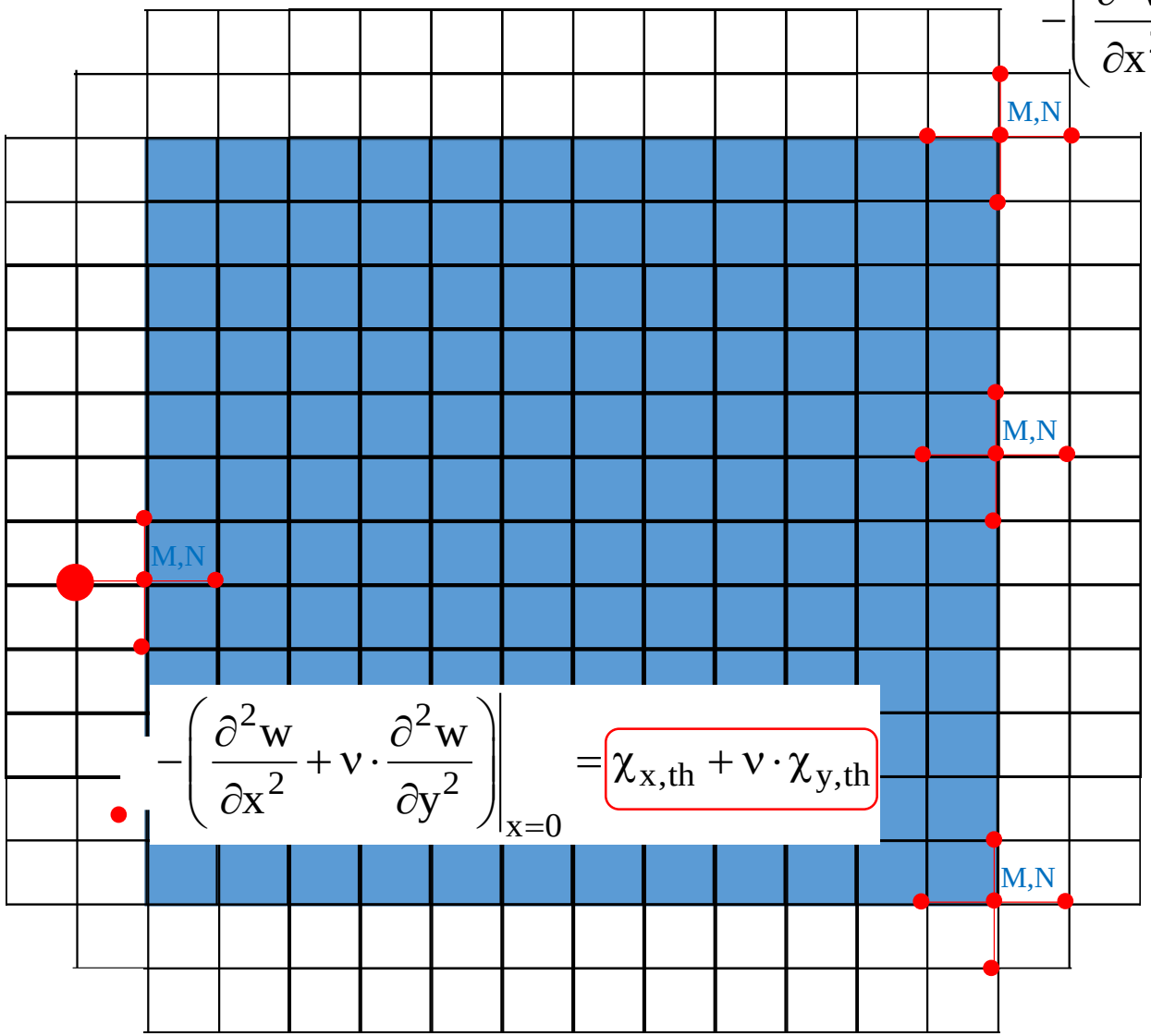
Field equation: spreadsheet implementation

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1				*0*	*500*	*1000*	*1500*	*2000*	*2500*	*3000*	*3500*	*4000*	*4500*	*5000*		
2				4.55E+00	4.20E+00	4.20E+00	4.26E+00	4.30E+00	4.30E+00	4.30E+00	4.26E+00	4.20E+00	4.20E+00	4.55E+00		
3			3.89E+00	3.47E+00	3.28E+00	3.33E+00	3.50E+00	3.68E+00	3.75E+00	3.68E+00	3.50E+00	3.33E+00	3.28E+00	3.46E+00	3.89E+00	
4	*0*	4.55E+00	3.47E+00	2.93E+00	2.77E+00	2.88E+00	3.13E+00	3.38E+00	3.48E+00	3.38E+00	3.13E+00	2.88E+00	2.77E+00	2.93E+00	3.46E+00	4.55E+00
5	*500*	4.20E+00	3.28E+00	2.77E+00	2.66E+00	2.86E+00	3.22E+00	3.55E+00	3.70E+00	3.55E+00	3.21E+00	2.86E+00	2.66E+00	2.77E+00	3.28E+00	4.20E+00
6	*1000*	4.20E+00	3.33E+00	2.88E+00	2.86E+00	3.18E+00	3.72E+00	4.22E+00	4.45E+00	4.22E+00	3.72E+00	3.18E+00	2.86E+00	2.88E+00	3.33E+00	4.19E+00
7	*1500*	4.26E+00	3.50E+00	3.13E+00	3.22E+00	3.72E+00	4.49E+00	5.27E+00	5.65E+00	5.27E+00	4.49E+00	3.72E+00	3.21E+00	3.13E+00	3.50E+00	4.26E+00
8	*2000*	4.30E+00	3.68E+00	3.38E+00	3.56E+00	4.22E+00	5.27E+00	6.42E+00	7.09E+00	6.42E+00	5.27E+00	4.22E+00	3.55E+00	3.38E+00	3.68E+00	4.30E+00
9	=(Load!D9/Data!$B$5*Data!$B$12^4-(-8*D8-8*E9-8*D10-8*C9+2*E8+2*E10+							7.09E+00	8.21E+00	7.09E+00	5.65E+00	4.45E+00	3.70E+00	3.48E+00	3.75E+00	4.30E+00
10	2*C10+2*C8+D7+F9+D11+B9))/(20+Data!$B$6/Data!$B$5*Data!$B$12^4)							6.42E+00	7.09E+00	6.42E+00	5.27E+00	4.22E+00	3.55E+00	3.38E+00	3.68E+00	4.30E+00
11	*3500*	4.26E+00	3.50E+00	3.13E+00	3.22E+00	3.72E+00	4.49E+00	5.27E+00	5.65E+00	5.27E+00	4.49E+00	3.72E+00	3.21E+00	3.13E+00	3.50E+00	4.26E+00
12	*4000*	4.20E+00	3.33E+00	2.88E+00	2.86E+00	3.18E+00	3.72E+00	4.22E+00	4.45E+00	4.22E+00	3.72E+00	3.18E+00	2.86E+00	2.88E+00	3.33E+00	4.19E+00
13	*4500*	4.20E+00	3.28E+00	2.77E+00	2.66E+00	2.86E+00	3.22E+00	3.55E+00	3.70E+00	3.55E+00	3.22E+00	2.86E+00	2.66E+00	2.77E+00	3.28E+00	4.20E+00
14	*5000*	4.55E+00	3.47E+00	2.93E+00	2.77E+00	2.88E+00	3.13E+00	3.38E+00	3.48E+00	3.38E+00	3.13E+00	2.88E+00	2.77E+00	2.93E+00	3.46E+00	4.55E+00
15			3.89E+00	3.47E+00	3.28E+00	3.33E+00	3.50E+00	3.68E+00	3.75E+00	3.68E+00	3.50E+00	3.33E+00	3.28E+00	3.46E+00	3.89E+00	
16				4.55E+00	4.20E+00	4.20E+00	4.26E+00	4.30E+00	4.30E+00	4.30E+00	4.26E+00	4.20E+00	4.20E+00	4.55E+00		

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1				*0*	*500*	*1000*	*1500*	*2000*	*2500*	*3000*	*3500*	*4000*	*4500*	*5000*		
2				4.55E+00	4.20E+00	4.20E+00	4.26E+00	4.30E+00	4.30E+00	4.30E+00	4.26E+00	4.20E+00	4.20E+00	4.55E+00		
3			3.89E+00	3.47E+00	3.28E+00	3.33E+00	3.50E+00	3.68E+00	3.75E+00	3.68E+00	3.50E+00	3.33E+00	3.28E+00	3.46E+00	3.89E+00	
4	=(Load!D4/Data!$B$5*Data!$B$12^4-(-8*D3-8*E4-8*D5-8*C4+2*E3+2*E5+2*							3.38E+00	3.48E+00	3.38E+00	3.13E+00	2.88E+00	2.77E+00	2.93E+00	3.46E+00	4.55E+00
5	C5+2*C3+D2+F4+D6+B4))/(20+Data!$B$6/Data!$B$5*Data!$B$12^4)							3.55E+00	3.70E+00	3.55E+00	3.21E+00	2.86E+00	2.66E+00	2.77E+00	3.28E+00	4.20E+00
6	*1000*	4.20E+00	3.33E+00	2.88E+00	2.86E+00	3.18E+00	3.72E+00	4.22E+00	4.45E+00	4.22E+00	3.72E+00	3.18E+00	2.86E+00	2.88E+00	3.33E+00	4.19E+00
7	*1500*	4.26E+00	3.50E+00	3.13E+00	3.22E+00	3.72E+00	4.49E+00	5.27E+00	5.65E+00	5.27E+00	4.49E+00	3.72E+00	3.21E+00	3.13E+00	3.50E+00	4.26E+00
8	*2000*	4.30E+00	3.68E+00	3.38E+00	3.56E+00	4.22E+00	5.27E+00	6.42E+00	7.09E+00	6.42E+00	5.27E+00	4.22E+00	3.55E+00	3.38E+00	3.68E+00	4.30E+00
9	*2500*	4.30E+00	3.75E+00	3.48E+00	3.70E+00	4.45E+00	5.65E+00	7.09E+00	8.21E+00	7.09E+00	5.65E+00	4.45E+00	3.70E+00	3.48E+00	3.75E+00	4.30E+00
10	*3000*	4.30E+00	3.68E+00	3.38E+00	3.56E+00	4.22E+00	5.27E+00	6.42E+00	7.09E+00	6.42E+00	5.27E+00	4.22E+00	3.55E+00	3.38E+00	3.68E+00	4.30E+00
11	*3500*	4.26E+00	3.50E+00	3.13E+00	3.22E+00	3.72E+00	4.49E+00	5.27E+00	5.65E+00	5.27E+00	4.49E+00	3.72E+00	3.21E+00	3.13E+00	3.50E+00	4.26E+00
12	*4000*	4.20E+00	3.33E+00	2.88E+00	2.86E+00	3.18E+00	3.72E+00	4.22E+00	4.45E+00	4.22E+00	3.72E+00	3.18E+00	2.86E+00	2.88E+00	3.33E+00	4.19E+00
13	*4500*	4.20E+00	3.28E+00	2.77E+00	2.66E+00	2.86E+00	3.22E+00	3.55E+00	3.70E+00	3.55E+00	3.22E+00	2.86E+00	2.66E+00	2.77E+00	3.28E+00	4.20E+00
14	*5000*	4.55E+00	3.47E+00	2.93E+00	2.77E+00	2.88E+00	3.13E+00	3.38E+00	3.48E+00	3.38E+00	3.13E+00	2.88E+00	2.77E+00	2.93E+00	3.46E+00	4.55E+00
15			3.89E+00	3.47E+00	3.28E+00	3.33E+00	3.50E+00	3.68E+00	3.75E+00	3.68E+00	3.50E+00	3.33E+00	3.28E+00	3.46E+00	3.89E+00	
16				4.55E+00	4.20E+00	4.20E+00	4.26E+00	4.30E+00	4.30E+00	4.30E+00	4.26E+00	4.20E+00	4.20E+00	4.55E+00		

Implementation

Boundary conditions: finite difference discretisation



$$-\left(\frac{\partial^2 w}{\partial x^2}+\nu\cdot\frac{\partial^2 w}{\partial y^2}\right)\Bigg|_{x=a}=\chi_{x,th}+\nu\cdot\chi_{y,th}$$

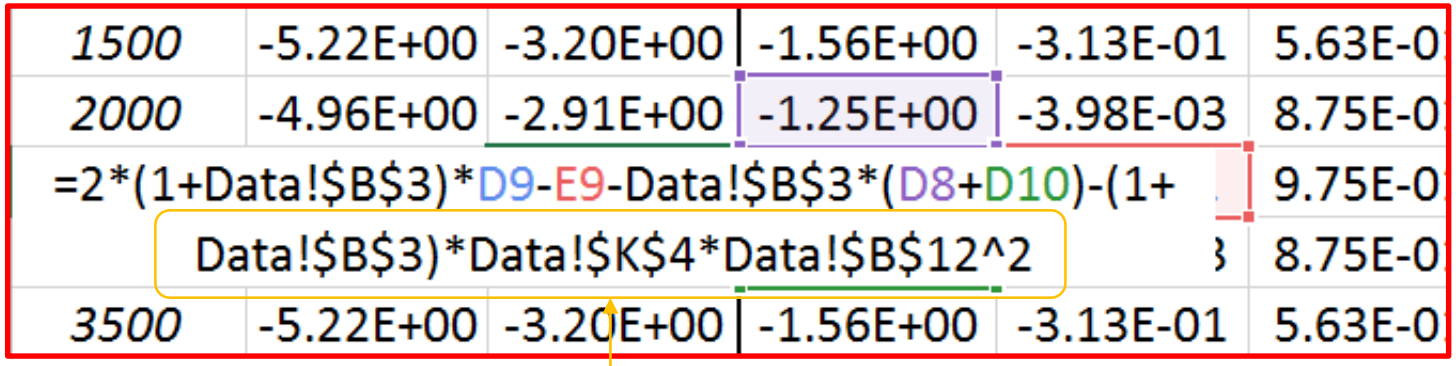
$$-\left(\frac{\partial^2 w}{\partial x^2}+\nu\cdot\frac{\partial^2 w}{\partial y^2}\right)\Bigg|_{x=0}=\chi_{x,th}+\nu\cdot\chi_{y,th}$$

$$\begin{aligned}&\nu\cdot w_{M,N+1}\\ &+w_{M-1,N}-2\cdot(1+\nu)\cdot w_{M,N}+w_{M+1,N}\\ &+\nu\cdot w_{M,N-1}\\ &+\left(\chi_{x,th}+\nu\cdot\chi_{y,th}\right)\cdot\Delta^2=0\end{aligned}$$

Implementation

Boundary conditions: spreadsheet implementation

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1				*0*	*500*	*1000*	*1500*	*2000*	*2500*	*3000*	*3500*	*4000*	*4500*	*5000*		
2				-7.63E+00	-6.50E+00	-5.71E+00	-5.22E+00	-4.96E+00	-4.89E+00	-4.96E+00	-5.22E+00	-5.71E+00	-6.50E+00	-7.63E+00		
3			-7.38E+00	-5.76E+00	-4.56E+00	-3.73E+00	-3.20E+00	-2.91E+00	-2.82E+00	-2.91E+00	-3.20E+00	-3.73E+00	-4.56E+00	-5.76E+00	-7.38E+00	
4	*0*	-7.63E+00	-5.76E+00	-4.17E+00	-2.96E+00	-2.10E+00	-1.56E+00	-1.25E+00	-1.16E+00	-1.25E+00	-1.56E+00	-2.10E+00	-2.96E+00	-4.17E+00	-5.76E+00	-7.63E+00
5	*500*	-6.50E+00	-4.56E+00	-2.96E+00	-1.74E+00	-8.73E-01	-3.13E-01	-3.98E-03	9.45E-02	-3.98E-03	-3.13E-01	-8.73E-01	-1.74E+00	-2.96E+00	-4.56E+00	-6.50E+00
6	*1000*	-5.71E+00	-3.73E+00	-2.10E+00	-8.73E-01	-2.03E-03	5.63E-01	8.75E-01	9.75E-01	8.75E-01	5.63E-01	-2.03E-03	-8.73E-01	-2.10E+00	-3.73E+00	-5.71E+00
7	*1500*	-5.22E+00	-3.20E+00	-1.56E+00	-3.13E-01	5.63E-01	1.13E+00	1.44E+00	1.54E+00	1.44E+00	1.13E+00	5.63E-01	-3.13E-01	-1.56E+00	-3.20E+00	-5.22E+00
8	*2000*	-4.96E+00	-2.91E+00	-1.25E+00	-3.98E-03	8.75E-01	1.44E+00	1.76E+00	1.86E+00	1.76E+00	1.44E+00	8.75E-01	-3.98E-03	-1.25E+00	-2.91E+00	-4.96E+00
9	=2*(1+Data!$B$3)*D9-E9-Data!$B$3*(D8+D10)-(1+					9.75E-01	1.54E+00	1.86E+00	1.96E+00	1.86E+00	1.54E+00	9.75E-01	9.45E-02	-1.16E+00	-2.82E+00	-4.89E+00
10	Data!$B$3)*Data!$K$4*Data!$B$12^2					8.75E-01	1.44E+00	1.76E+00	1.86E+00	1.76E+00	1.44E+00	8.75E-01	-3.98E-03	-1.25E+00	-2.91E+00	-4.96E+00
11	*3500*	-5.22E+00	-3.20E+00	-1.56E+00	-3.13E-01	5.63E-01	1.13E+00	1.44E+00	1.54E+00	1.44E+00	1.13E+00	5.63E-01	-3.13E-01	-1.56E+00	-3.20E+00	-5.22E+00
12	*4000*	-5.71E+00	-3.73E+00	-2.10E+00	-8.73E-01	-2.03E-03	5.63E-01	8.75E-01	9.75E-01	8.75E-01	5.63E-01	-2.03E-03	-8.73E-01	-2.10E+00	-3.73E+00	-5.71E+00
13	*4500*	-6.50E+00	-4.56E+00	-2.96E+00	-1.74E+00	-8.73E-01	-3.13E-01	-3.98E-03	9.45E-02	-3.98E-03	-3.13E-01	-8.73E-01	-1.74E+00	-2.96E+00	-4.56E+00	-6.50E+00
14	*5000*	-7.63E+00	-5.76E+00	-4.17E+00	-2.96E+00	-2.10E+00	-1.56E+00	-1.25E+00	-1.16E+00	-1.25E+00	-1.56E+00	-2.10E+00	-2.96E+00	-4.17E+00	-5.76E+00	-7.63E+00
15			-7.38E+00	-5.76E+00	-4.56E+00	-3.73E+00	-3.20E+00	-2.91E+00	-2.82E+00	-2.91E+00	-3.20E+00	-3.73E+00	-4.56E+00	-5.76E+00	-7.38E+00	
16				-7.63E+00	-6.50E+00	-5.71E+00	-5.22E+00	-4.96E+00	-4.89E+00	-4.96E+00	-5.22E+00	-5.71E+00	-6.50E+00	-7.63E+00		



$$\left(\chi_{x,th} + \nu \cdot \chi_{y,th}\right) \cdot \Delta^2 = -(1+\nu) \cdot \frac{\alpha_c \Delta T}{s}$$

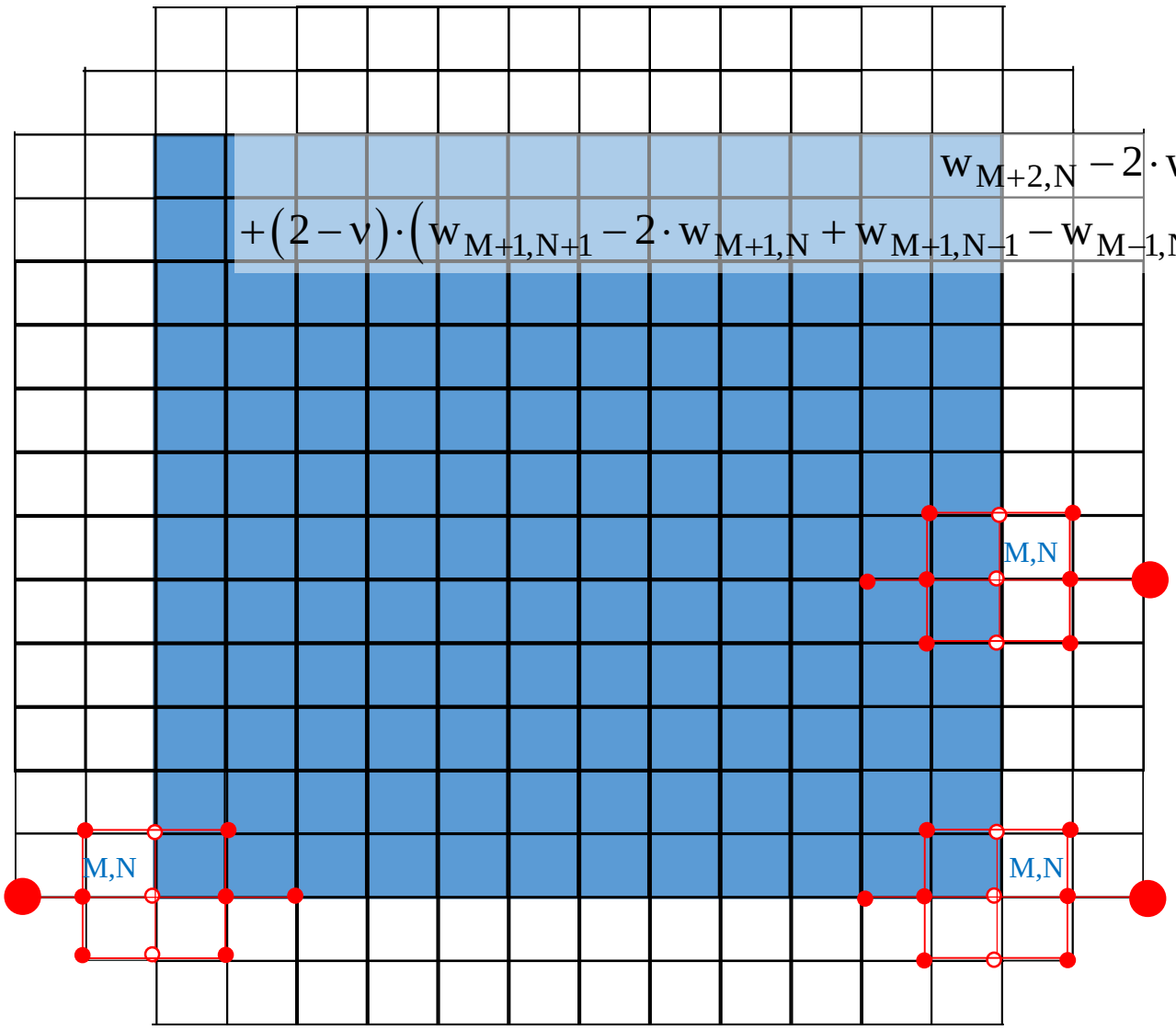
Implementation

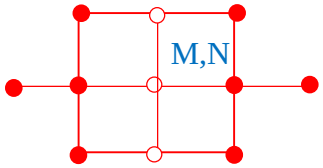
Boundary conditions: finite difference discretisation

$$-D\cdot\left(\frac{\partial^3 w}{\partial x^3}+(2-\nu)\cdot\frac{\partial^3 w}{\partial x\partial y^2}\right)_{x=a}=0$$

$$w_{M+2,N}-2\cdot w_{M+1,N}+2\cdot w_{M-1,N}-w_{M-2,N}+$$
$$+(2-\nu)\cdot\left(w_{M+1,N+1}-2\cdot w_{M+1,N}+w_{M+1,N-1}-w_{M-1,N+1}+2\cdot w_{M-1,N}-w_{M-1,N-1}\right)=0$$

$$-D\cdot\left(\frac{\partial^3 w}{\partial x^3}+(2-\nu)\cdot\frac{\partial^3 w}{\partial x\partial y^2}\right)_{x=0}=0$$





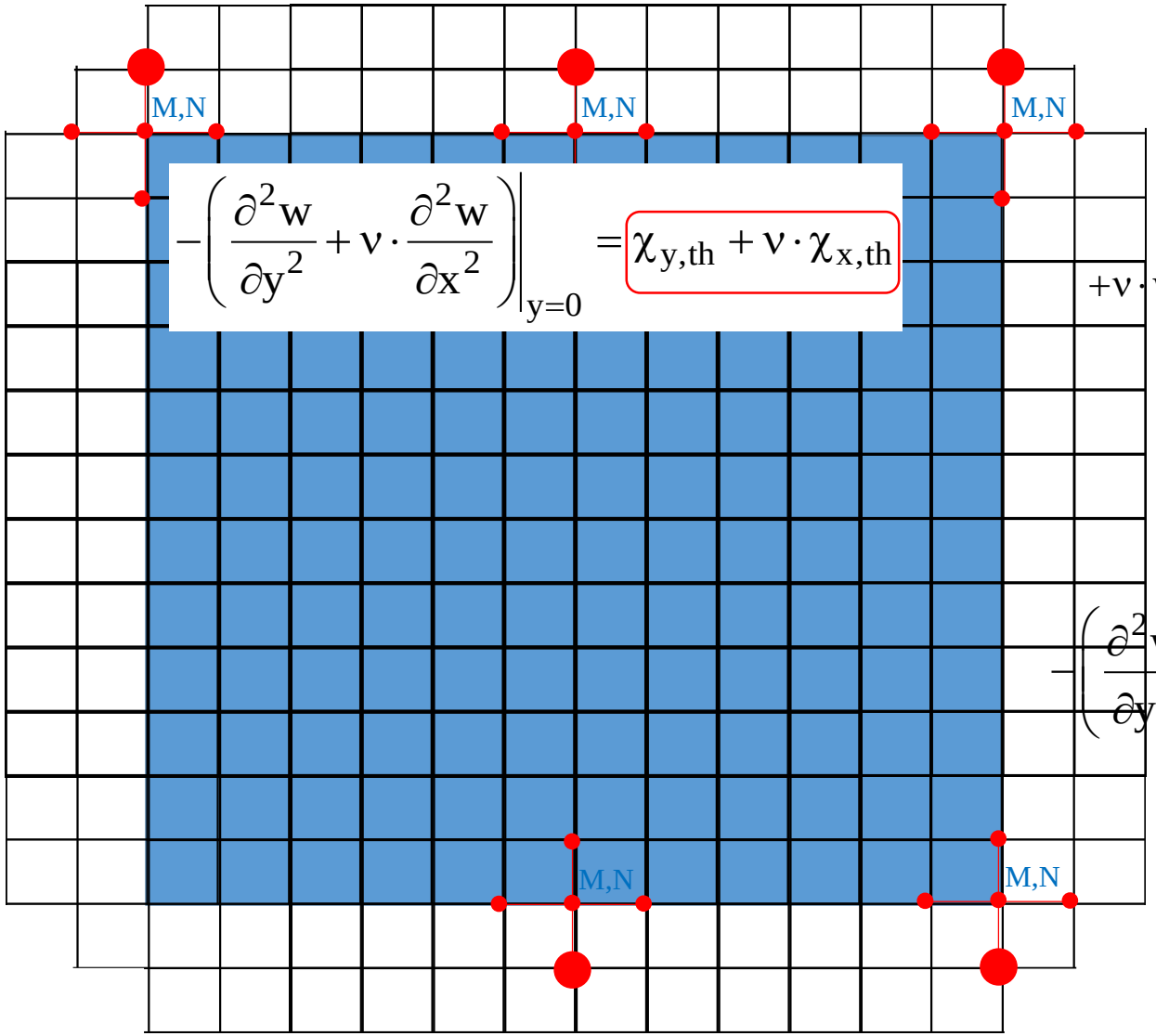
Implementation

Boundary conditions: spreadsheet implementation

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1				*0*	*500*	*1000*	*1500*	*2000*	*2500*	*3000*	*3500*	*4000*	*4500*	*5000*		
2				-7.63E+00	-6.50E+00	-5.71E+00	-5.22E+00	-4.96E+00	-4.89E+00	-4.96E+00	-5.22E+00	-5.71E+00	-6.50E+00	-7.63E+00		
3			-7.38E+00	-5.76E+00	-4.56E+00	-3.73E+00	-3.20E+00	-2.91E+00	-2.82E+00	-2.91E+00	-3.20E+00	-3.73E+00	-4.56E+00	-5.76E+00	-7.38E+00	
4	*0*	-7.63E+00	-5.76E+00	-4.17E+00	-2.96E+00	-2.10E+00	-1.56E+00	-1.25E+00	-1.16E+00	-1.25E+00	-1.56E+00	-2.10E+00	-2.96E+00	-4.17E+00	-5.76E+00	-7.63E+00
5	*500*	-6.50E+00	-4.56E+00	-2.96E+00	-1.74E+00	-8.73E-01	-3.13E-01	-3.98E-03	9.45E-02	-3.98E-03	-3.13E-01	-8.73E-01	-1.74E+00	-2.96E+00	-4.56E+00	-6.50E+00
6	*1000*	-5.71E+00	-3.73E+00	-2.10E+00	-8.73E-01	-2.03E-03	5.63E-01	8.75E-01	9.75E-01	8.75E-01	5.63E-01	-2.03E-03	-8.73E-01	-2.10E+00	-3.73E+00	-5.71E+00
7	*1500*	-5.22E+00	-3.20E+00	-1.56E+00	-3.13E-01	5.63E-01	1.13E+00	1.44E+00	1.54E+00	1.44E+00	1.13E+00	5.63E-01	-3.13E-01	-1.56E+00	-3.20E+00	-5.22E+00
8	*2000*	-4.96E+00	-2.91E+00	-1.25E+00	-3.98E-03	8.75E-01	1.44E+00	1.76E+00	1.86E+00	1.76E+00	1.44E+00	8.75E-01	-3.98E-03	-1.25E+00	-2.91E+00	-4.96E+00
9	*2500*	-4.89E+00	-2.82E+00	-1.16E+00	9.45E-02	9.75E-01	1.54E+00	1.86E+00	1.96E+00	1.86E+00	1.54E+00	9.75E-01	9.45E-02	-1.16E+00	-2.82E+00	-4.89E+00
10	=(F10+E11-C11+E9-C9-4*(E10-			-1.25E+00	-3.98E-03	8.75E-01	1.44E+00	1.76E+00	1.86E+00	1.76E+00	1.44E+00	8.75E-01	-3.98E-03	-1.25E+00	-2.91E+00	-4.96E+00
11	C10))+(1-Data!$B$3)*(E11-C11+			-1.56E+00	-3.13E-01	5.63E-01	1.13E+00	1.44E+00	1.54E+00	1.44E+00	1.13E+00	5.63E-01	-3.13E-01	-1.56E+00	-3.20E+00	-5.22E+00
12	E9-C9-2*(E10-C10))			-2.10E+00	-8.73E-01	-2.03E-03	5.63E-01	8.75E-01	9.75E-01	8.75E-01	5.63E-01	-2.03E-03	-8.73E-01	-2.10E+00	-3.73E+00	-5.71E+00
13	*4500*	-6.50E+00	-4.56E+00	-2.96E+00	-1.74E+00	-8.73E-01	-3.13E-01	-3.98E-03	9.45E-02	-3.98E-03	-3.13E-01	-8.73E-01	-1.74E+00	-2.96E+00	-4.56E+00	-6.50E+00
14	*5000*	-7.63E+00	-5.76E+00	-4.17E+00	-2.96E+00	-2.10E+00	-1.56E+00	-1.25E+00	-1.16E+00	-1.25E+00	-1.56E+00	-2.10E+00	-2.96E+00	-4.17E+00	-5.76E+00	-7.63E+00
15			-7.38E+00	-5.76E+00	-4.56E+00	-3.73E+00	-3.20E+00	-2.91E+00	-2.82E+00	-2.91E+00	-3.20E+00	-3.73E+00	-4.56E+00	-5.76E+00	-7.38E+00	
16				-7.63E+00	-6.50E+00	-5.71E+00	-5.22E+00	-4.96E+00	-4.89E+00	-4.96E+00	-5.22E+00	-5.71E+00	-6.50E+00	-7.63E+00		

Implementation

Boundary conditions: finite difference discretisation



$$-\left(\frac{\partial^2 w}{\partial y^2}+\nu\cdot\frac{\partial^2 w}{\partial x^2}\right)\Bigg|_{y=0}=\chi_{y,th}+\nu\cdot\chi_{x,th}$$

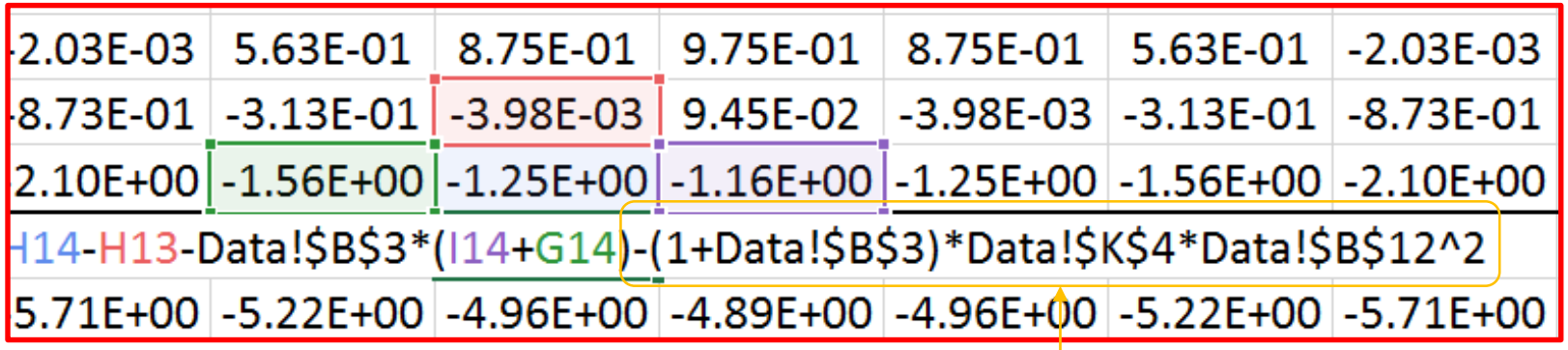
$$w_{M+1,N}+\nu\cdot w_{M,N-1}-2\cdot(1+\nu)\cdot w_{M,N}+\nu\cdot w_{M,N+1}+w_{M-1,N}+\left(\chi_{y,th}+\nu\cdot\chi_{x,th}\right)\cdot\Delta^2=0$$

$$-\left(\frac{\partial^2 w}{\partial y^2}+\nu\cdot\frac{\partial^2 w}{\partial x^2}\right)\Bigg|_{y=b}=\chi_{y,th}+\nu\cdot\chi_{x,th}$$

Implementation

Boundary conditions: spreadsheet implementation

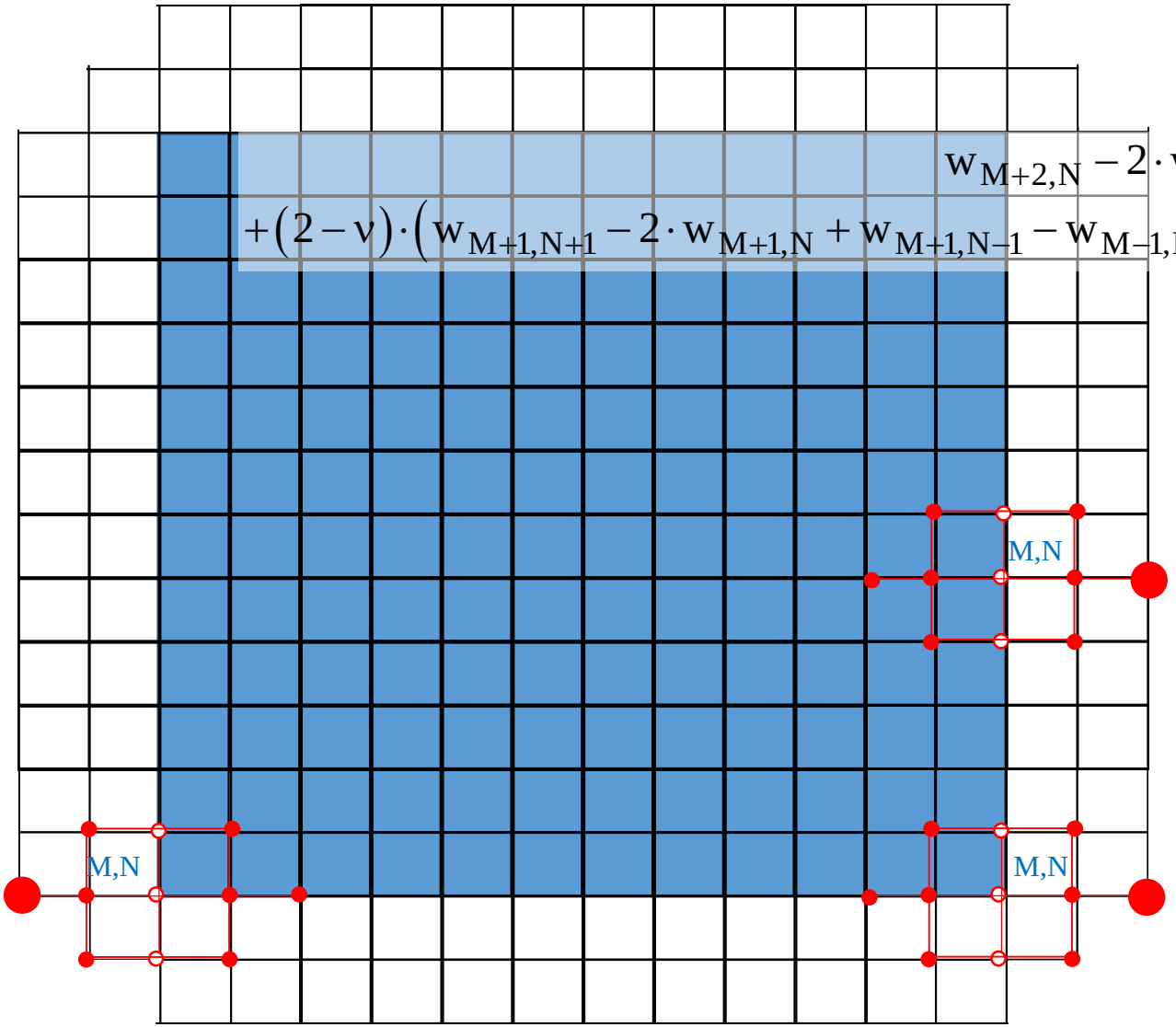
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1				*0*	*500*	*1000*	*1500*	*2000*	*2500*	*3000*	*3500*	*4000*	*4500*	*5000*		
2				-7.63E+00	-6.50E+00	-5.71E+00	-5.22E+00	-4.96E+00	-4.89E+00	-4.96E+00	-5.22E+00	-5.71E+00	-6.50E+00	-7.63E+00		
3			-7.38E+00	-5.76E+00	-4.56E+00	-3.73E+00	-3.20E+00	-2.91E+00	-2.82E+00	-2.91E+00	-3.20E+00	-3.73E+00	-4.56E+00	-5.76E+00	-7.38E+00	
4	*0*	-7.63E+00	-5.76E+00	-4.17E+00	-2.96E+00	-2.10E+00	-1.56E+00	-1.25E+00	-1.16E+00	-1.25E+00	-1.56E+00	-2.10E+00	-2.96E+00	-4.17E+00	-5.76E+00	-7.63E+00
5	*500*	-6.50E+00	-4.56E+00	-2.96E+00	-1.74E+00	-8.73E-01	-3.13E-01	-3.98E-03	9.45E-02	-3.98E-03	-3.13E-01	-8.73E-01	-1.74E+00	-2.96E+00	-4.56E+00	-6.50E+00
6	*1000*	-5.71E+00	-3.73E+00	-2.10E+00	-8.73E-01	-2.03E-03	5.63E-01	8.75E-01	9.75E-01	8.75E-01	5.63E-01	-2.03E-03	-8.73E-01	-2.10E+00	-3.73E+00	-5.71E+00
7	*1500*	-5.22E+00	-3.20E+00	-1.56E+00	-3.13E-01	5.63E-01	1.13E+00	1.44E+00	1.54E+00	1.44E+00	1.13E+00	5.63E-01	-3.13E-01	-1.56E+00	-3.20E+00	-5.22E+00
8	*2000*	-4.96E+00	-2.91E+00	-1.25E+00	-3.98E-03	8.75E-01	1.44E+00	1.76E+00	1.86E+00	1.76E+00	1.44E+00	8.75E-01	-3.98E-03	-1.25E+00	-2.91E+00	-4.96E+00
9	*2500*	-4.89E+00	-2.82E+00	-1.16E+00	9.45E-02	9.75E-01	1.54E+00	1.86E+00	1.96E+00	1.86E+00	1.54E+00	9.75E-01	9.45E-02	-1.16E+00	-2.82E+00	-4.89E+00
10	*3000*	-4.96E+00	-2.91E+00	-1.25E+00	-3.98E-03	8.75E-01	1.44E+00	1.76E+00	1.86E+00	1.76E+00	1.44E+00	8.75E-01	-3.98E-03	-1.25E+00	-2.91E+00	-4.96E+00
11	*3500*	-5.22E+00	-3.20E+00	-1.56E+00	-3.13E-01	5.63E-01	1.13E+00	1.44E+00	1.54E+00	1.44E+00	1.13E+00	5.63E-01	-3.13E-01	-1.56E+00	-3.20E+00	-5.22E+00
12	*4000*	-5.71E+00	-3.73E+00	-2.10E+00	-8.73E-01	-2.03E-03	5.63E-01	8.75E-01	9.75E-01	8.75E-01	5.63E-01	-2.03E-03	-8.73E-01	-2.10E+00	-3.73E+00	-5.71E+00
13	*4500*	-6.50E+00	-4.56E+00	-2.96E+00	-1.74E+00	-8.73E-01	-3.13E-01	-3.98E-03	9.45E-02	-3.98E-03	-3.13E-01	-8.73E-01	-1.74E+00	-2.96E+00	-4.56E+00	-6.50E+00
14	*5000*	-7.63E+00	-5.76E+00	-4.17E+00	-2.96E+00	-2.10E+00	-1.56E+00	-1.25E+00	-1.16E+00	-1.25E+00	-1.56E+00	-2.10E+00	-2.96E+00	-4.17E+00	-5.76E+00	-7.63E+00
15			-7.38E+00	=2*(1+Data!$B$3)*H14-H13-Data!$B$3*(I14+G14)-(1+Data!$B$3)*Data!$K$4*Data!$B$12^2									-4.56E+00	-5.76E+00	-7.38E+00	
16				-7.63E+00	-6.50E+00	-5.71E+00	-5.22E+00	-4.96E+00	-4.89E+00	-4.96E+00	-5.22E+00	-5.71E+00	-6.50E+00	-7.63E+00		



$$\left(\chi_{x,th} + \nu \cdot \chi_{y,th}\right) \cdot \Delta^2 = -(1+\nu) \cdot \frac{\alpha_c \Delta T}{s}$$

Implementation

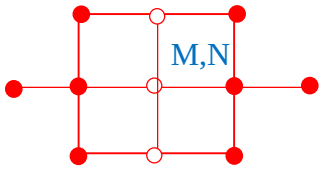
Boundary conditions: finite difference discretisation



$$-D\cdot\left(\frac{\partial^3 w}{\partial x^3}+(2-\nu)\cdot\frac{\partial^3 w}{\partial x\partial y^2}\right)_{x=a}=0$$

$$w_{M+2,N}-2\cdot w_{M+1,N}+2\cdot w_{M-1,N}-w_{M-2,N}+$$
$$+(2-\nu)\cdot\left(w_{M+1,N+1}-2\cdot w_{M+1,N}+w_{M+1,N-1}-w_{M-1,N+1}+2\cdot w_{M-1,N}-w_{M-1,N-1}\right)=0$$

$$-D\cdot\left(\frac{\partial^3 w}{\partial x^3}+(2-\nu)\cdot\frac{\partial^3 w}{\partial x\partial y^2}\right)_{x=0}=0$$



Implementation

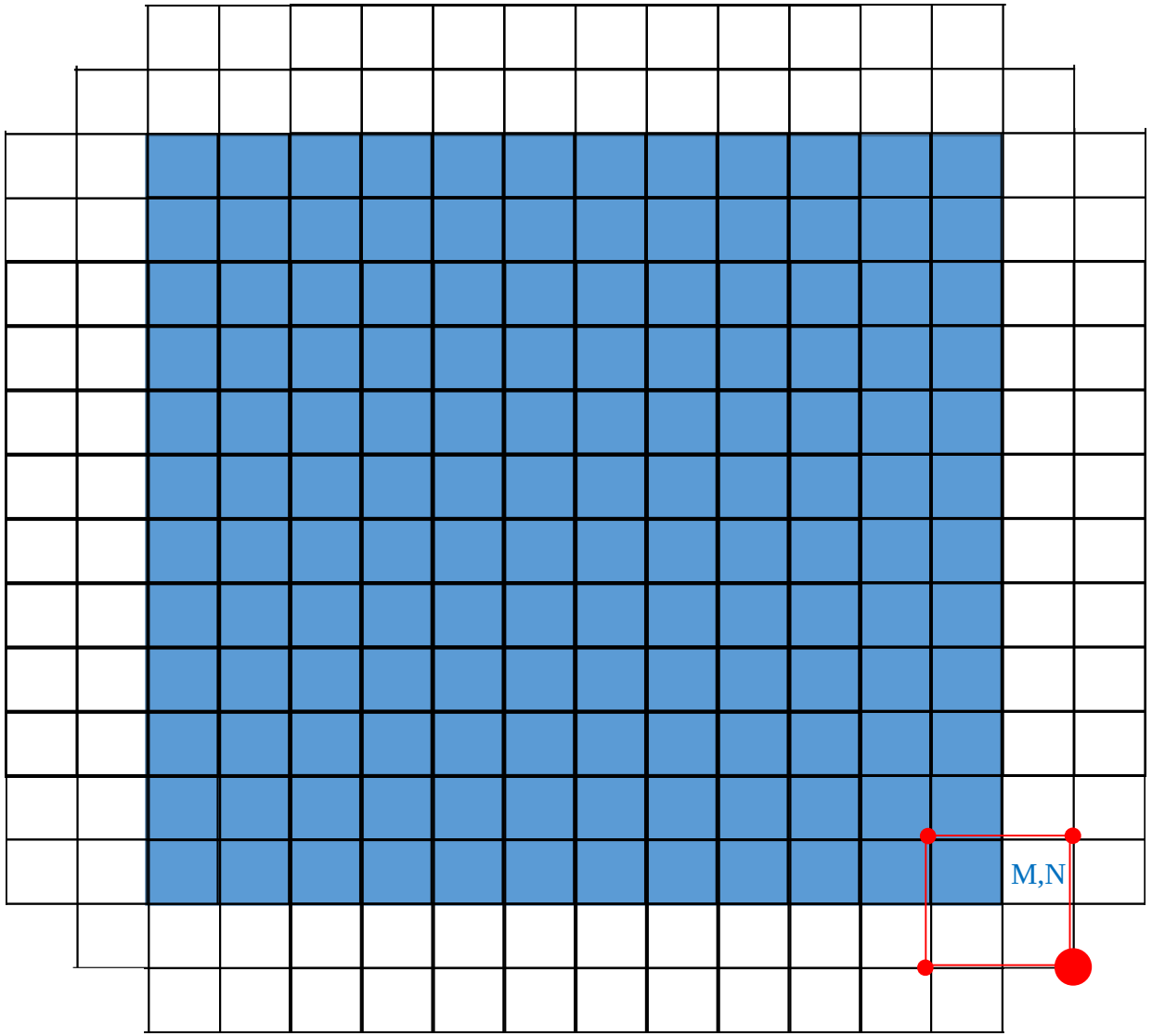
Boundary conditions: spreadsheet implementation

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1				*0*	*500*	*1000*	*1500*	*2000*	*2500*	*3000*	*3500*	*4000*	*4500*	*5000*		
2				-7.63E+00	-6.50E+00	-5.71E+00	-5.22E+00	-4.96E+00	-4.89E+00	-4.96E+00	-5.22E+00	-5.71E+00	-6.50E+00	-7.63E+00		
3			-7.38E+00	-5.76E+00	-4.56E+00	-3.73E+00	-3.20E+00	-2.91E+00	-2.82E+00	-2.91E+00	-3.20E+00	-3.73E+00	-4.56E+00	-5.76E+00	-7.38E+00	
4	*0*	-7.63E+00	-5.76E+00	-4.17E+00	-2.96E+00	-2.10E+00	-1.56E+00	-1.25E+00	-1.16E+00	-1.25E+00	-1.56E+00	-2.10E+00	-2.96E+00	-4.17E+00	-5.76E+00	-7.63E+00
5	*500*	-6.50E+00	-4.56E+00	-2.96E+00	-1.74E+00	-8.73E-01	-3.13E-01	-3.98E-03	9.45E-02	-3.98E-03	-3.13E-01	-8.73E-01	-1.74E+00	-2.96E+00	-4.56E+00	-6.50E+00
6	=(F6+E7-C7+E5-C5-4*		(E6-C6))+(	-2.10E+00	-8.73E-01	-2.03E-03	5.63E-01	8.75E-01	9.75E-01	8.75E-01	5.63E-01	-2.03E-03	-8.73E-01	-2.10E+00	-3.73E+00	-5.71E+00
7	1-Data!$B$3)*(E7-C7+E5-C5-2*(			-1.56E+00	-3.13E-01	5.63E-01	1.13E+00	1.44E+00	1.54E+00	1.44E+00	1.13E+00	5.63E-01	-3.13E-01	-1.56E+00	-3.20E+00	-5.22E+00
8		E6-C6))		-1.25E+00	-3.98E-03	8.75E-01	1.44E+00	1.76E+00	1.86E+00	1.76E+00	1.44E+00	8.75E-01	-3.98E-03	-1.25E+00	-2.91E+00	-4.96E+00
9	*2500*	-4.89E+00	-2.82E+00	-1.16E+00	9.45E-02	9.75E-01	1.54E+00	1.86E+00	1.96E+00	1.86E+00	1.54E+00	9.75E-01	9.45E-02	-1.16E+00	-2.82E+00	-4.89E+00
10	*3000*	-4.96E+00	-2.91E+00	-1.25E+00	-3.98E-03	8.75E-01	1.44E+00	1.76E+00	1.86E+00	1.76E+00	1.44E+00	8.75E-01	-3.98E-03	-1.25E+00	-2.91E+00	-4.96E+00
11	*3500*	-5.22E+00	-3.20E+00	-1.56E+00	-3.13E-01	5.63E-01	1.13E+00	1.44E+00	1.54E+00	1.44E+00	1.13E+00	5.63E-01	-3.13E-01	-1.56E+00	-3.20E+00	-5.22E+00
12	*4000*	-5.71E+00	-3.73E+00	-2.10E+00	-8.73E-01	-2.03E-03	5.63E-01	8.75E-01	9.75E-01	8.75E-01	5.63E-01	-2.03E-03	-8.73E-01	-2.10E+00	-3.73E+00	-5.71E+00
13	*4500*	-6.50E+00	-4.56E+00	-2.96E+00	-1.74E+00	-8.73E-01	-3.13E-01	-3.98E-03	9.45E-02	-3.98E-03	-3.13E-01	-8.73E-01	-1.74E+00	-2.96E+00	-4.56E+00	-6.50E+00
14	*5000*	-7.63E+00	-5.76E+00	-4.17E+00	-2.96E+00	-2.10E+00	-1.56E+00	-1.25E+00	-1.16E+00	-1.25E+00	-1.56E+00	-2.10E+00	-2.96E+00	-4.17E+00	-5.76E+00	-7.63E+00
15			-7.38E+00	-5.76E+00	-4.56E+00	-3.73E+00	-3.20E+00	-2.91E+00	-2.82E+00	-2.91E+00	-3.20E+00	-3.73E+00	-4.56E+00	-5.76E+00	-7.38E+00	
16				-7.63E+00	-6.50E+00	-5.71E+00	-5.22E+00	-4.96E+00	-4.89E+00	-4.96E+00	-5.22E+00	-5.71E+00	-6.50E+00	-7.63E+00		

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1				*0*	*500*	*1000*	*1500*	*2000*	*2500*	*3000*	*3500*	*4000*	*4500*	*5000*		
2				-7.63E+00	-6.50E+00	-5.71E+00	-5.22E+00	-4.96E+00	-4.89E+00	-4.96E+00	-5.22E+00	-5.71E+00	-6.50E+00	-7.63E+00		
3			-7.38E+00	-5.76E+00	-4.56E+00	-3.73E+00	-3.20E+00	-2.91E+00	-2.82E+00	-2.91E+00	-3.20E+00	-3.73E+00	-4.56E+00	-5.76E+00	-7.38E+00	
4	*0*	-7.63E+00	-5.76E+00	-4.17E+00	-2.96E+00	-2.10E+00	-1.56E+00	-1.25E+00	-1.16E+00	-1.25E+00	-1.56E+00	-2.10E+00	-2.96E+00	-4.17E+00	-5.76E+00	-7.63E+00
5	*500*	-6.50E+00	-4.56E+00	-2.96E+00	-1.74E+00	-8.73E-01	-3.13E-01	-3.98E-03	9.45E-02	-3.98E-03	-3.13E-01	-8.73E-01	-1.74E+00	-2.96E+00	-4.56E+00	-6.50E+00
6	*1000*	-5.71E+00	-3.73E+00	-2.10E+00	-8.73E-01	-2.03E-03	5.63E-01	8.75E-01	9.75E-01	8.75E-01	5.63E-01	-2.03E-03	-8.73E-01	-2.10E+00	-3.73E+00	-5.71E+00
7	*1500*	-5.22E+00	-3.20E+00	-1.56E+00	-3.13E-01	5.63E-01	1.13E+00	1.44E+00	1.54E+00	1.44E+00	1.13E+00	5.63E-01	-3.13E-01	-1.56E+00	-3.20E+00	-5.22E+00
8	*2000*	-4.96E+00	-2.91E+00	-1.25E+00	-3.98E-03	8.75E-01	1.44E+00	1.76E+00	1.86E+00	1.76E+00	1.44E+00	8.75E-01	-3.98E-03	-1.25E+00	-2.91E+00	-4.96E+00
9	*2500*	-4.89E+00	-2.82E+00	-1.16E+00	9.45E-02	9.75E-01	1.54E+00	1.86E+00	1.96E+00	1.86E+00	1.54E+00	9.75E-01	9.45E-02	-1.16E+00	-2.82E+00	-4.89E+00
10	*3000*	-4.96E+00	-2.91E+00	-1.25E+00	-3.98E-03	8.75E-01	1.44E+00	1.76E+00	1.86E+00	1.76E+00	1.44E+00	8.75E-01	-3.98E-03	-1.25E+00	-2.91E+00	-4.96E+00
11	*3500*	-5.22E+00	-3.20E+00	-1.56E+00	-3.13E-01	5.63E-01	1.13E+00	1.44E+00	1.54E+00	1.44E+00	1.13E+00	5.63E-01	-3.13E-01	-1.56E+00	-3.20E+00	4*(O11-
12	*4000*	-5.71E+00	-3.73E+00	-2.10E+00	-8.73E-01	-2.03E-03	5.63E-01	8.75E-01	9.75E-01	8.75E-01	5.63E-01	-2.03E-03	-8.73E-01	-2.10E+00	-3.73E+00	M11))-(1-
13	*4500*	-6.50E+00	-4.56E+00	-2.96E+00	-1.74E+00	-8.73E-01	-3.13E-01	-3.98E-03	9.45E-02	-3.98E-03	-3.13E-01	-8.73E-01	-1.74E+00	-2.96E+00	-4.56E+00	Data!
14	*5000*	-7.63E+00	-5.76E+00	-4.17E+00	-2.96E+00	-2.10E+00	-1.56E+00	-1.25E+00	-1.16E+00	-1.25E+00	-1.56E+00	-2.10E+00	-2.96E+00	-4.17E+00	-5.76E+00	$B$3)*(
15			-7.38E+00	-5.76E+00	-4.56E+00	-3.73E+00	-3.20E+00	-2.91E+00	-2.82E+00	-2.91E+00	-3.20E+00	-3.73E+00	-4.56E+00	-5.76E+00	-7.38E+00	O12-M12+
16				-7.63E+00	-6.50E+00	-5.71E+00	-5.22E+00	-4.96E+00	-4.89E+00	-4.96E+00	-5.22E+00	-5.71E+00	-6.50E+00	-7.63E+00		O10-M10-

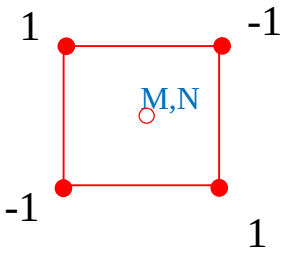
Implementation

Boundary conditions: finite difference discretisation



$$-(1-\nu)\cdot D\cdot \left.\frac{\partial^2 w}{\partial x \partial y}\right|_{\substack{x=a \\ y=b}} = 0$$

$$w_{M+1,N+1} - w_{M+1,N-1} + \\ -w_{M-1,N+1} + w_{M-1,N-1} = 0$$



Implementation

Boundary conditions: spreadsheet implementation

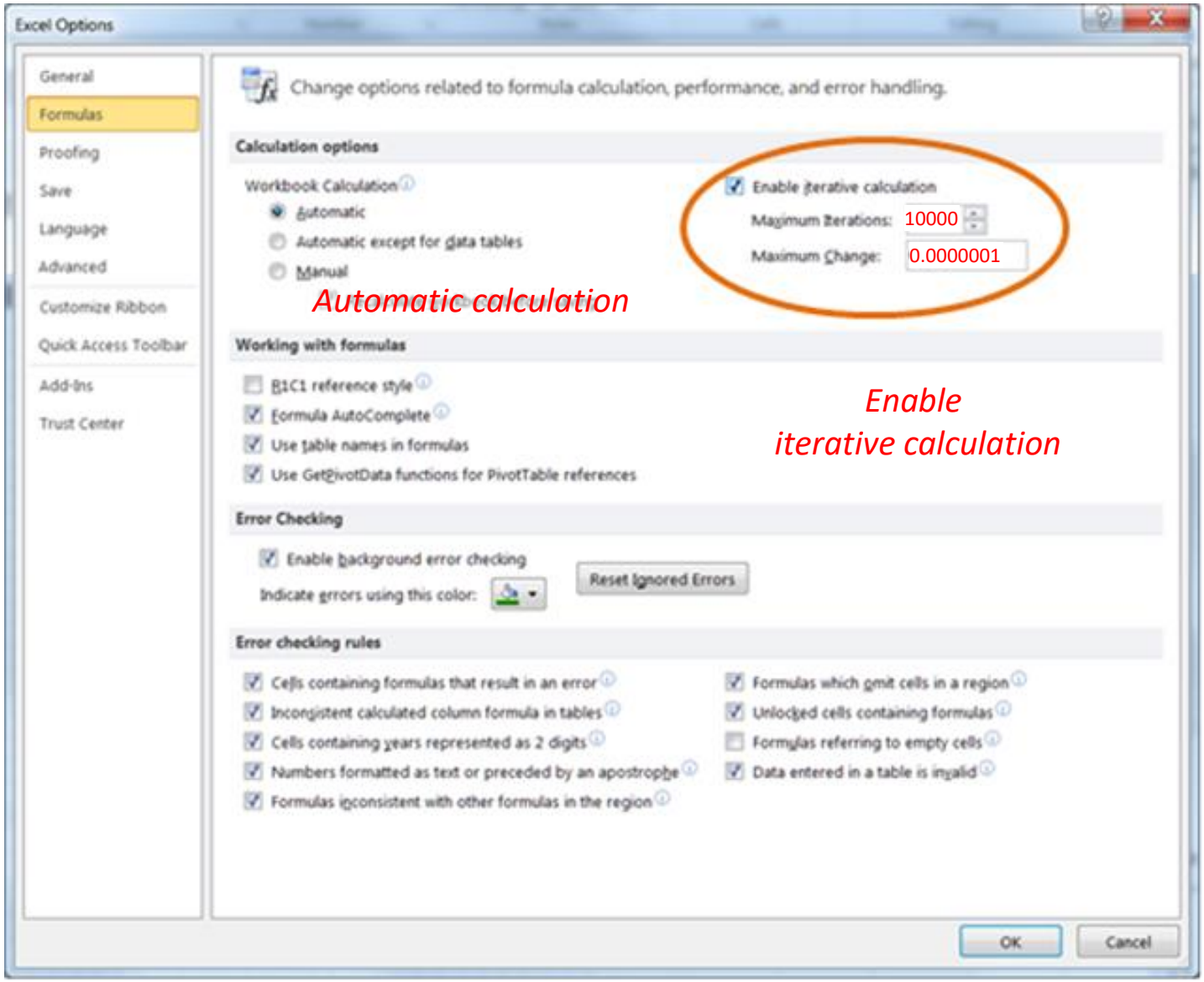
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1				*0*	*500*	*1000*	*1500*	*2000*	*2500*	*3000*	*3500*	*4000*	*4500*	*5000*		
2				-7.63E+00	-6.50E+00	-5.71E+00	-5.22E+00	-4.96E+00	-4.89E+00	-4.96E+00	-5.22E+00	-5.71E+00	-6.50E+00	-7.63E+00		
3			-7.38E+00	-5.76E+00	-4.56E+00	-3.73E+00	-3.20E+00	-2.91E+00	-2.82E+00	-2.91E+00	-3.20E+00	-3.73E+00	-4.56E+00	-5.76E+00	-7.38E+00	
4	*0*	-7.63E+00	-5.76E+00	-4.17E+00	-2.96E+00	-2.10E+00	-1.56E+00	-1.25E+00	-1.16E+00	-1.25E+00	-1.56E+00	-2.10E+00	-2.96E+00	-4.17E+00	-5.76E+00	-7.63E+00
5	*500*	-6.50E+00	-4.56E+00	-2.96E+00	-1.74E+00	-8.73E-01	-3.13E-01	-3.98E-03	9.45E-02	-3.98E-03	-3.13E-01	-8.73E-01	-1.74E+00	-2.96E+00	-4.56E+00	-6.50E+00
6	*1000*	-5.71E+00	-3.73E+00	-2.10E+00	-8.73E-01	-2.03E-03	5.63E-01	8.75E-01	9.75E-01	8.75E-01	5.63E-01	-2.03E-03	-8.73E-01	-2.10E+00	-3.73E+00	-5.71E+00
7	*1500*	-5.22E+00	-3.20E+00	-1.56E+00	-3.13E-01	5.63E-01	1.13E+00	1.44E+00	1.54E+00	1.44E+00	1.13E+00	5.63E-01	-3.13E-01	-1.56E+00	-3.20E+00	-5.22E+00
8	*2000*	-4.96E+00	-2.91E+00	-1.25E+00	-3.98E-03	8.75E-01	1.44E+00	1.76E+00	1.86E+00	1.76E+00	1.44E+00	8.75E-01	-3.98E-03	-1.25E+00	-2.91E+00	-4.96E+00
9	*2500*	-4.89E+00	-2.82E+00	-1.16E+00	9.45E-02	9.75E-01	1.54E+00	1.86E+00	1.96E+00	1.86E+00	1.54E+00	9.75E-01	9.45E-02	-1.16E+00	-2.82E+00	-4.89E+00
10	*3000*	-4.96E+00	-2.91E+00	-1.25E+00	-3.98E-03	8.75E-01	1.44E+00	1.76E+00	1.86E+00	1.76E+00	1.44E+00	8.75E-01	-3.98E-03	-1.25E+00	-2.91E+00	-4.96E+00
11	*3500*	-5.22E+00	-3.20E+00	-1.56E+00	-3.13E-01	5.63E-01	1.13E+00	1.44E+00	1.54E+00	1.44E+00	1.13E+00	5.63E-01	-3.13E-01	-1.56E+00	-3.20E+00	-5.22E+00
12	*4000*	-5.71E+00	-3.73E+00	-2.10E+00	-8.73E-01	-2.03E-03	5.63E-01	8.75E-01	9.75E-01	8.75E-01	5.63E-01	-2.03E-03	-8.73E-01	-2.10E+00	-3.73E+00	-5.71E+00
13	*4500*	-6.50E+00	-4.56E+00	-2.96E+00	-1.74E+00	-8.73E-01	-3.13E-01	-3.98E-03	9.45E-02	-3.98E-03	-3.13E-01	-8.73E-01	-1.74E+00	-2.96E+00	-4.56E+00	-6.50E+00
14	*5000*	-7.63E+00	-5.76E+00	-4.17E+00	-2.96E+00	-2.10E+00	-1.56E+00	-1.25E+00	-1.16E+00	-1.25E+00	-1.56E+00	-2.10E+00	-2.96E+00	-4.17E+00	-5.76E+00	-7.63E+00
15			-7.38E+00	-5.76E+00	-4.56E+00	-3.73E+00	-3.20E+00	-2.91E+00	-2.82E+00	-2.91E+00	-3.20E+00	-3.73E+00	-4.56E+00	=M15+O13-M13		
16				-7.63E+00	-6.50E+00	-5.71E+00	-5.22E+00	-4.96E+00	-4.89E+00	-4.96E+00	-5.22E+00	-5.71E+00	-6.50E+00	-7.63E+00		

$$w_{M+1,N+1} = w_{M+1,N-1} + w_{M-1,N+1} - w_{M-1,N-1}$$

-2.03E-03	-8.73E-01	-2.10E+00	-3.73E+00	-5.71E+00
-8.73E-01	-1.74E+00	-2.96E+00	-4.56E+00	-6.50E+00
-2.10E+00	-2.96E+00	-4.17E+00	-5.76E+00	-7.63E+00
-3.73E+00	-4.56E+00	=M15+O13-M13		
-5.71E+00	-6.50E+00	-7.63E+00		

Implementation

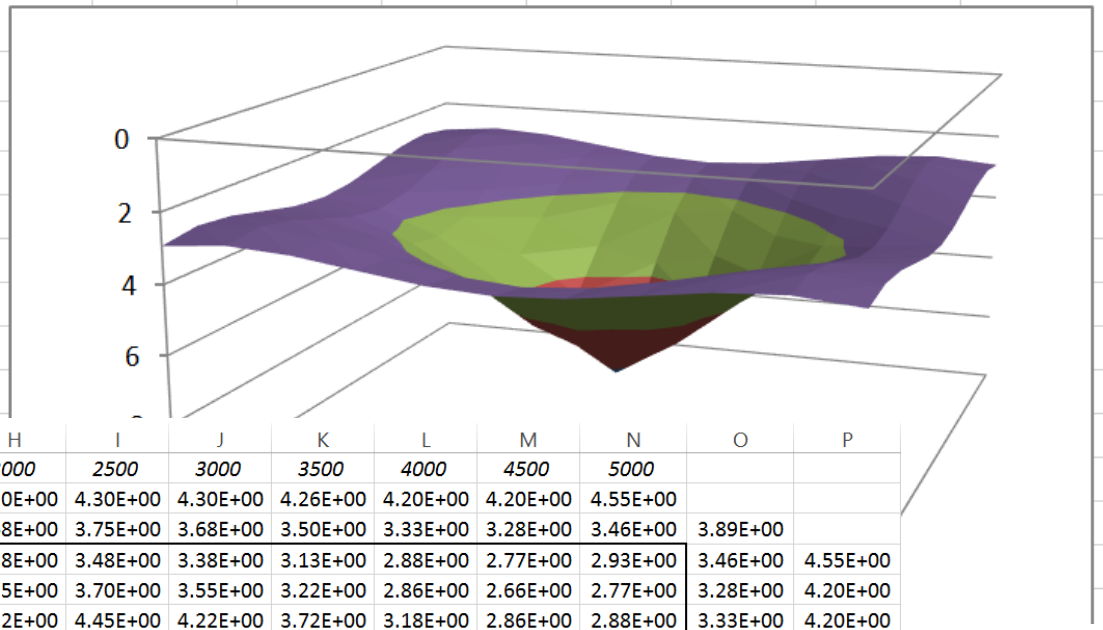
Iterative Solution



Implementation

Iterative Solution: displacements

E=	31476	MPa	**Self-weight multiplier**			**Center-point force**			**Thermal effects**	
s=	200	mm	γ_g=	0.00		F=	1000.00	kN	Δt=	25
ν=	0.2		g=	5.00	kN/m				α_c=	1.20E-05
γ=	25.0	kN/m^3							χ^*=	-1.50E-06
D=	2.186E+10	Nmm								
k_0=	0.01	N/mm^3								
L_x=	5000	mm								
L_y=	5000	mm								
n=	10									
Δ=	500	mm								



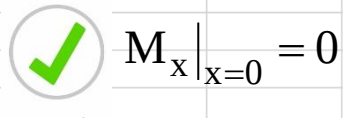
	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1			*0*	*500*	*1000*	*1500*	*2000*	*2500*	*3000*	*3500*	*4000*	*4500*	*5000*		
2			4.55E+00	4.20E+00	4.20E+00	4.26E+00	4.30E+00	4.30E+00	4.30E+00	4.26E+00	4.20E+00	4.20E+00	4.55E+00		
3		3.89E+00	3.46E+00	3.28E+00	3.33E+00	3.50E+00	3.68E+00	3.75E+00	3.68E+00	3.50E+00	3.33E+00	3.28E+00	3.46E+00	3.89E+00	
4	4.55E+00	3.46E+00	2.93E+00	2.77E+00	2.88E+00	3.13E+00	3.38E+00	3.48E+00	3.38E+00	3.13E+00	2.88E+00	2.77E+00	2.93E+00	3.46E+00	4.55E+00
5	4.20E+00	3.28E+00	2.77E+00	2.66E+00	2.86E+00	3.22E+00	3.55E+00	3.70E+00	3.55E+00	3.22E+00	2.86E+00	2.66E+00	2.77E+00	3.28E+00	4.20E+00
6	4.20E+00	3.33E+00	2.88E+00	2		3.72E+00	4.22E+00	4.45E+00	4.22E+00	3.72E+00	3.18E+00	2.86E+00	2.88E+00	3.33E+00	4.20E+00
7	4.26E+00	3.50E+00	3.13E+00	3		4.49E+00	5.27E+00	5.65E+00	5.27E+00	4.49E+00	3.72E+00	3.22E+00	3.13E+00	3.50E+00	4.26E+00
8	4.30E+00	3.68E+00	3.38E+00	3		5.27E+00	6.42E+00	7.09E+00	6.42E+00	5.27E+00	4.22E+00	3.55E+00	3.38E+00	3.68E+00	4.30E+00
9	4.30E+00	3.75E+00	3.48E+00	3		5.65E+00	7.09E+00	8.21E+00	7.09E+00	5.65E+00	4.45E+00	3.70E+00	3.48E+00	3.75E+00	4.30E+00
10	4.30E+00	3.68E+00	3.38E+00	3.55E+00	4.22E+00	5.27E+00	6.42E+00	7.09E+00	6.42E+00	5.27E+00	4.22E+00	3.55E+00	3.38E+00	3.68E+00	4.30E+00
11	4.26E+00	3.50E+00	3.13E+00	3.22E+00	3.72E+00	4.49E+00	5.27E+00	5.65E+00	5.27E+00	4.49E+00	3.72E+00	3.22E+00	3.13E+00	3.50E+00	4.26E+00
12	4.20E+00	3.33E+00	2.88E+00	2.86E+00	3.18E+00	3.72E+00	4.22E+00	4.45E+00	4.22E+00	3.72E+00	3.18E+00	2.86E+00	2.88E+00	3.33E+00	4.20E+00
13	4.20E+00	3.28E+00	2.77E+00	2.66E+00	2.86E+00	3.22E+00	3.55E+00	3.70E+00	3.55E+00	3.22E+00	2.86E+00	2.66E+00	2.77E+00	3.28E+00	4.20E+00
14	4.55E+00	3.46E+00	2.93E+00	2.77E+00	2.88E+00	3.13E+00	3.38E+00	3.48E+00	3.38E+00	3.13E+00	2.88E+00	2.77E+00	2.93E+00	3.46E+00	4.55E+00
15		3.89E+00	3.46E+00	3.28E+00	3.33E+00	3.50E+00	3.68E+00	3.75E+00	3.68E+00	3.50E+00	3.33E+00	3.28E+00	3.46E+00	3.89E+00	
16			4.55E+00	4.20E+00	4.20E+00	4.26E+00	4.30E+00	4.30E+00	4.30E+00	4.26E+00	4.20E+00	4.20E+00	4.55E+00		

$$\overline{w}_{M,N}$$

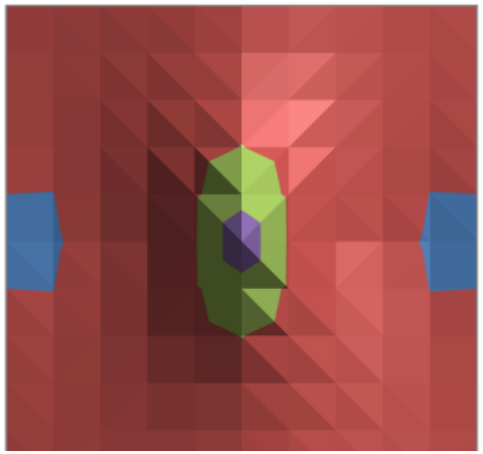
Implementation

Solution: bending moments M_x

			0	500	1000	1500	2000	2500	3000	3500	4000	4500	5000
0			0.00	8814.93	19589.01	32180.91	43474.15	48410.02	43474.15	32180.91	19589.01	8814.93	0.00
500			0.00	7360.98	18927.22	33692.34	48003.60	54860.98	48003.60	33692.34	18927.22	7360.98	0.00
1000			0.00	5693.96	17990.35	36515.33	57671.68	70370.28	57671.68	36515.33	17990.35	5693.96	0.00
1500			0.00	3071.15	15601.52	38989.67	72637.10	##########	72637.10	38989.67	15601.52	3071.15	0.00
2000			0.00	-222.71	11218.75	37942.59	88670.98	##########	88670.98	37942.59	11218.75	-222.71	0.00
2500			0.00	-2201.56	7385.30	32242.14	90949.85	##########	90949.85	32242.14	7385.30	-2201.56	0.00
3000			0.00	-222.71	11218.75	37942.59	88670.98	##########	88670.98	37942.59	11218.75	-222.71	0.00
3500			0.00	3071.15	15601.52	38989.67	72637.10	##########	72637.10	38989.67	15601.52	3071.15	0.00
4000			0.00	5693.96	17990.35	36515.33	57671.68	70370.28	57671.68	36515.33	17990.35	5693.96	0.00
4500			0.00	7360.98	18927.22	33692.34	48003.60	54860.98	48003.60	33692.34	18927.22	7360.98	0.00
5000			0.00	8814.93	19589.01	32180.91	43474.15	48410.02	43474.15	32180.91	19589.01	8814.93	0.00



$$M_x\big|_{x=0} = 0$$



$$M_x\big|_{x=a} = 0$$

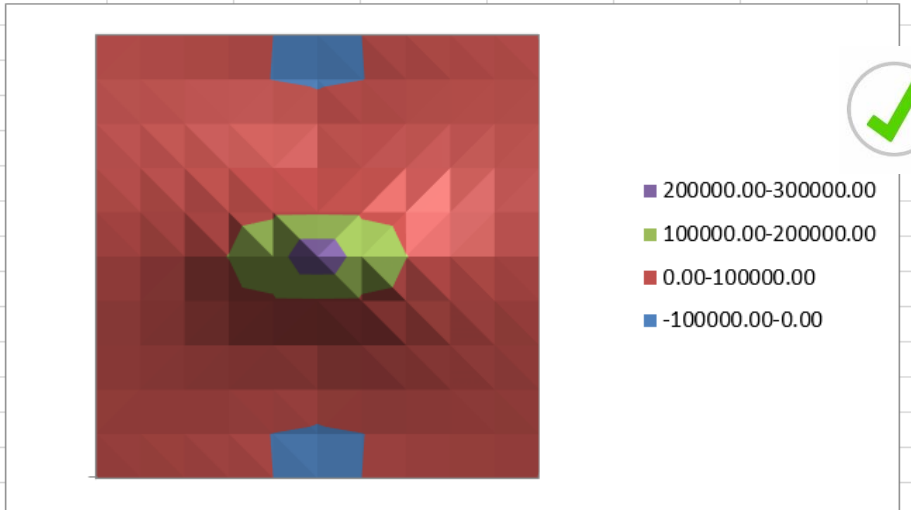
$$M_{x_{M,N}} = -D \cdot \left(\frac{\bar{w}_{M+1,N} - 2\bar{w}_{M,N} + \bar{w}_{M-1,N}}{\Delta^2} + \nu \cdot \frac{\bar{w}_{M,N+1} - 2\bar{w}_{M,N} + \bar{w}_{M,N-1}}{\Delta^2} \right) - D \cdot (1+\nu) \cdot \frac{\alpha_c \Delta T}{s}$$

Implementation

Solution: bending moments M_y

$$M_y\big|_{y=0} = 0$$

	0	500	1000	1500	2000	2500	3000	3500	4000	4500	5000
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
500	8814.93	7360.98	5693.96	3071.15	-222.71	-2201.56	-222.71	3071.15	5693.96	7360.98	8814.93
1000	19589.01	18927.22	17990.35	15601.52	11218.75	7385.30	11218.75	15601.52	17990.35	18927.22	19589.01
1500	32180.91	33692.34	36515.33	38989.67	37942.59	32242.14	37942.59	38989.67	36515.33	33692.34	32180.91
2000	43474.15	48003.60	57671.68	72637.10	88670.98	90949.85	88670.98	72637.10	57671.68	48003.60	43474.15
2500	48410.02	54860.98	70370.28	##########	##########	##########	##########	##########	70370.28	54860.98	48410.02
3000	43474.15	48003.60	57671.68	72637.10	88670.98	90949.85	88670.98	72637.10	57671.68	48003.60	43474.15
3500	32180.91	33692.34	36515.33	38989.67	37942.59	32242.14	37942.59	38989.67	36515.33	33692.34	32180.91
4000	19589.01	18927.22	17990.35	15601.52	11218.75	7385.30	11218.75	15601.52	17990.35	18927.22	19589.01
4500	8814.93	7360.98	5693.96	3071.15	-222.71	-2201.56	-222.71	3071.15	5693.96	7360.98	8814.93
5000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00



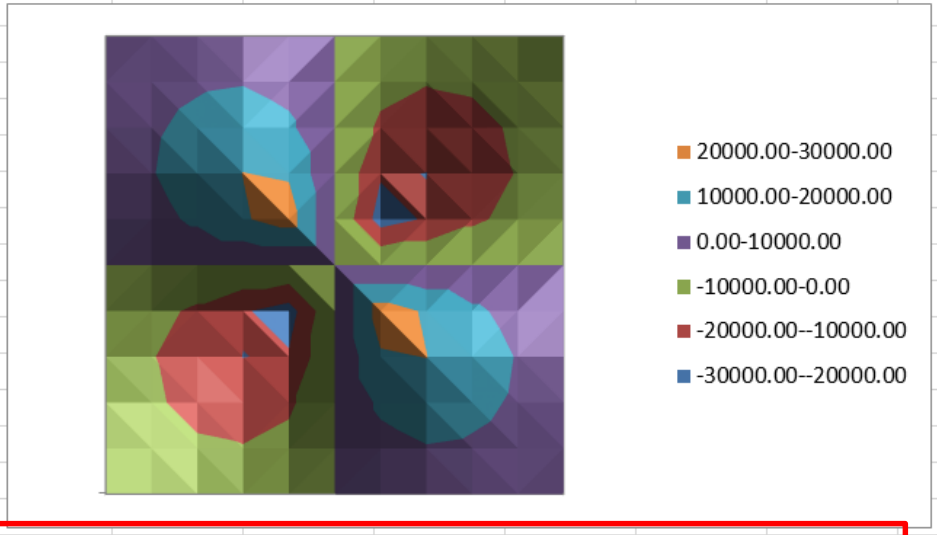
$$M_y\big|_{y=b} = 0$$

$$M_{y_{M,N}} = -D \cdot \left(\frac{\bar{w}_{M,N+1} - 2\bar{w}_{M,N} + \bar{w}_{M,N-1}}{\Delta^2} + \nu \cdot \frac{\bar{w}_{M+1,N} - 2\bar{w}_{M,N} + \bar{w}_{M-1,N}}{\Delta^2} \right) - D \cdot (1+\nu) \cdot \frac{\alpha_c \Delta T}{s}$$

Implementation

Solution: torsional moments M_{xy}

			0	500	1000	1500	2000	2500	3000	3500	4000	4500	5000
0			0.00	-3894.77	-5706.87	-6061.65	-4100.98	0.00	4100.98	6061.65	5706.87	3894.77	0.00
500			-3894.77	-6149.82	-8671.31	-9501.02	-6757.70	0.00	6757.70	9501.02	8671.31	6149.82	3894.77
1000			-5706.87	-8671.31	-12639.24	-15009.31	-11860.55	0.00	11860.55	15009.31	12639.24	8671.31	5706.87
1500			-6061.65	-9501.02	-15009.31	-20152.59	-19000.71	0.00	19000.71	20152.59	15009.31	9501.02	6061.65
2000			-4100.98	-6757.70	-11860.55	-19000.71	-24404.85	0.00	24404.85	19000.71	11860.55	6757.70	4100.98
2500			0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3000			4100.98	6757.70	11860.55	19000.71	24404.85	0.00	-24404.85	-19000.71	-11860.55	-6757.70	-4100.98
3500			6061.65	9501.02	15009.31	20152.59	19000.71	0.00	-19000.71	-20152.59	-15009.31	-9501.02	-6061.65
4000			5706.87	8671.31	12639.24	15009.31	11860.55	0.00	-11860.55	-15009.31	-12639.24	-8671.31	-5706.87
4500			3894.77	6149.82	8671.31	9501.02	6757.70	0.00	-6757.70	-9501.02	-8671.31	-6149.82	-3894.77
5000			0.00	3894.77	5706.87	6061.65	4100.98	0.00	-4100.98	-6061.65	-5706.87	-3894.77	0.00



$$M_{xy_{M,N}} = -D \cdot (1-\nu) \cdot \left(\frac{\bar{w}_{M+1,N+1} - \bar{w}_{M+1,N-1} + \bar{w}_{M-1,N-1} - \bar{w}_{M-1,N+1}}{4\Delta^2} \right)$$

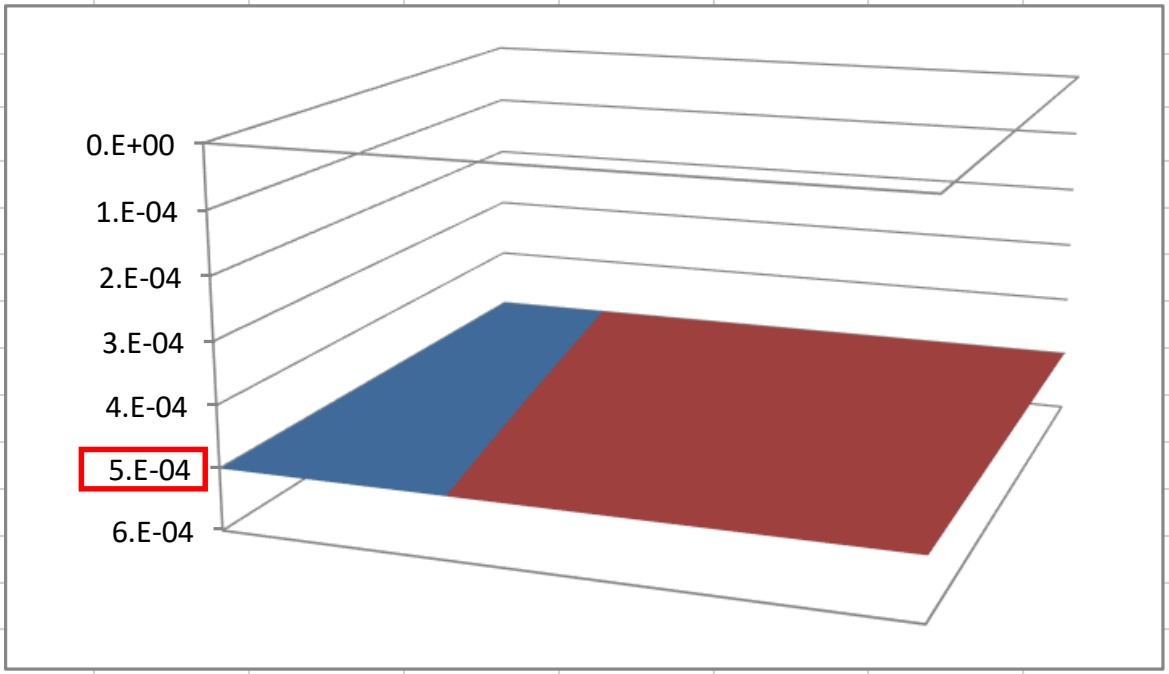
Overview

- ✓ Clarifications about lecture 1;
- ✓ Addition of thermal effects;
- ✓ Implementation in MS Excel;
- ✓ Examples and comparisons.

Examples and Comparison

Load Case 1: self-weight

E=	31476	MPa	**Self-weight multiplier**			**Center-point force**			**Thermal effects**	
s=	200	mm	γ_g=	1.00		F=	0.00	kN	Δt=	0
ν=	0.2		g=	5.00	kN/m^2				α_c=	1.20E-05
γ=	25.0	kN/m^3							χ*=	0.00E+00
D=	2.186E+10	Nmm								
k_0=	0.01	N/mm^3								
L_x=	5000	mm								
L_y=	5000	mm								
n=	10									
Δ=	500	mm								

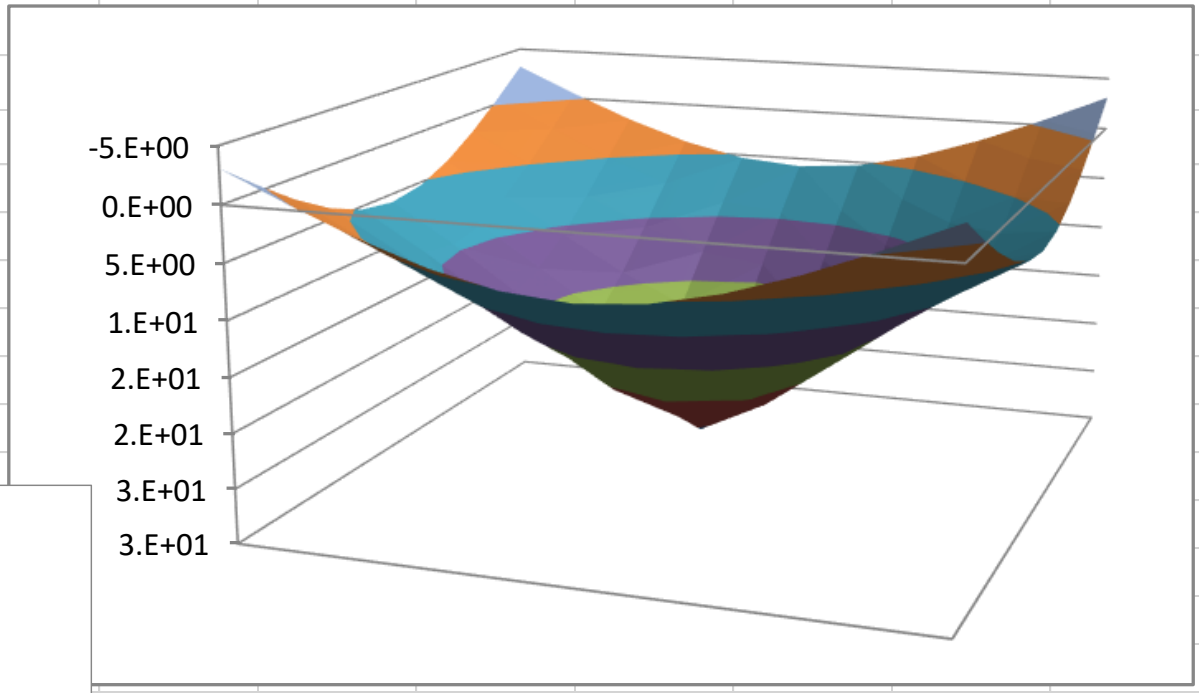


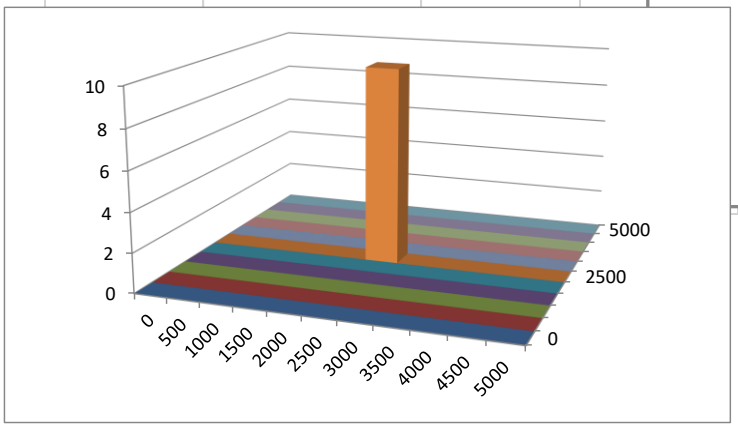
$$w = \frac{g}{k_0} = \frac{5 \cdot 10^{-6}\ \mathrm{N/mm^2}}{0.01\ \mathrm{N/mm^3}} = 5 \cdot 10^{-4}\ \mathrm{mm}$$

Examples and Comparison

Load Case 2: concentrated force in the center point

E=	31476	MPa	**Self-weight multiplier**			**Center-point force**			**Thermal effects**	
s=	200	mm	γ_g=	0.00		F=	2500.00	kN	Δt=	0
ν=	0.2		g=	5.00	kN/m^2				α_c=	1.20E-05
γ=	25.0	kN/m^3							χ*=	0.00E+00
D=	2.186E+10	Nmm								
k_0=	0.01	N/mm^3								
L_x=	5000	mm								
L_y=	5000	mm								
n=	10									
Δ=	500	mm								



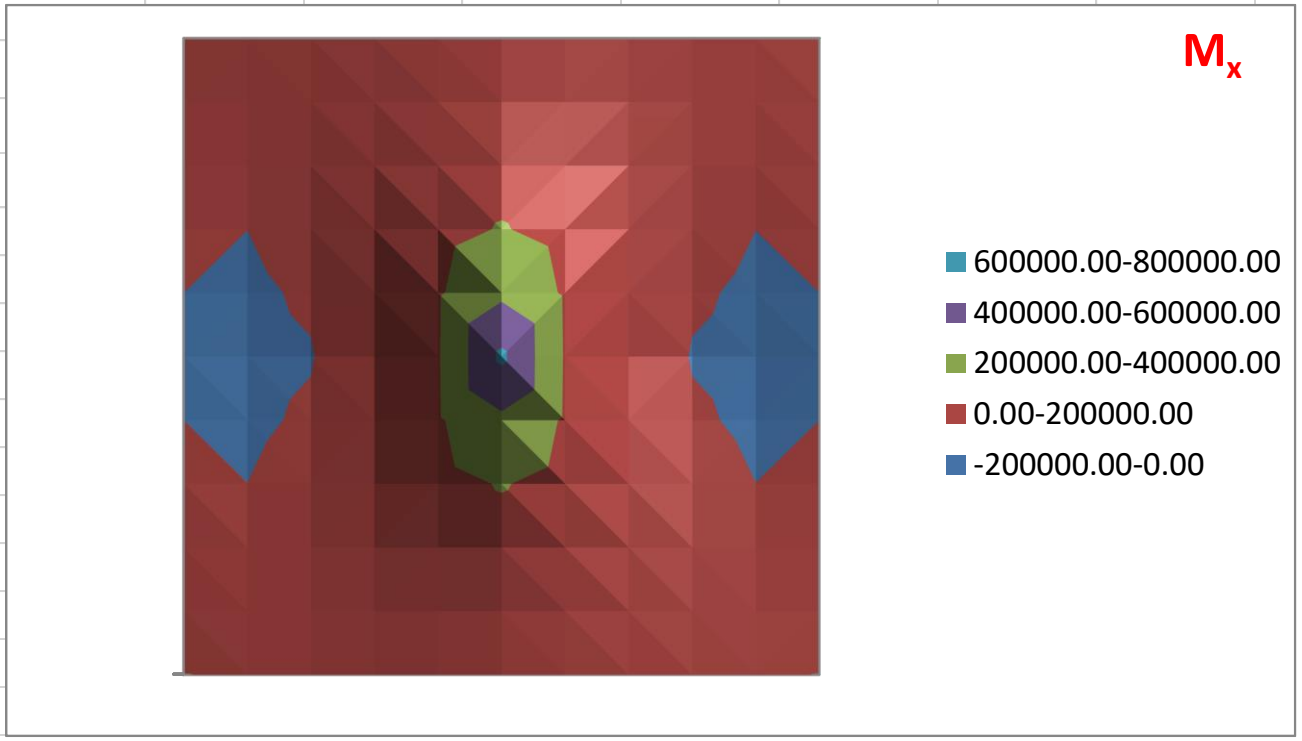


w_{max}=25.41 mm

Examples and Comparison

Load Case 2: concentrated force in the center point

$E=$	31476	MPa	**Self-weight multiplier**			**Center-point force**			**Thermal effects**	
$s=$	200	mm	$\gamma_g=$	0.00		$F=$	2500.00	kN	$\Delta t=$	0
$\nu=$	0.2		$g=$	5.00	kN/m^2				$\alpha_c=$	1.20E-05
$\gamma=$	25.0	kN/m^3							$\chi^*=$	0.00E+00
$D=$	2.186E+10	Nmm								
$k_0=$	0.01	N/mm^3								
$L_x=$	5000	mm								
$L_y=$	5000	mm								
$n=$	10									
$\Delta=$	500	mm								

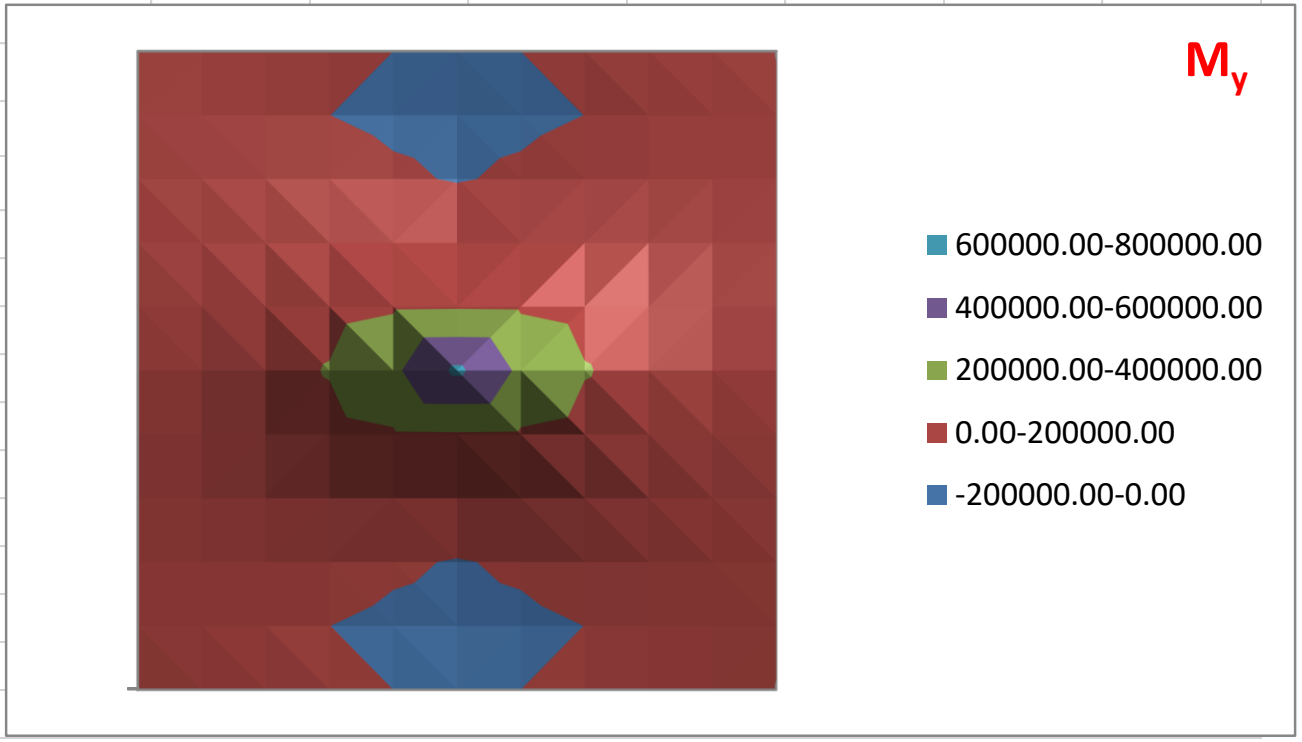


$M_{x,max}$=63870.1 Nmm/mm

Examples and Comparison

Load Case 2: concentrated force in the center point

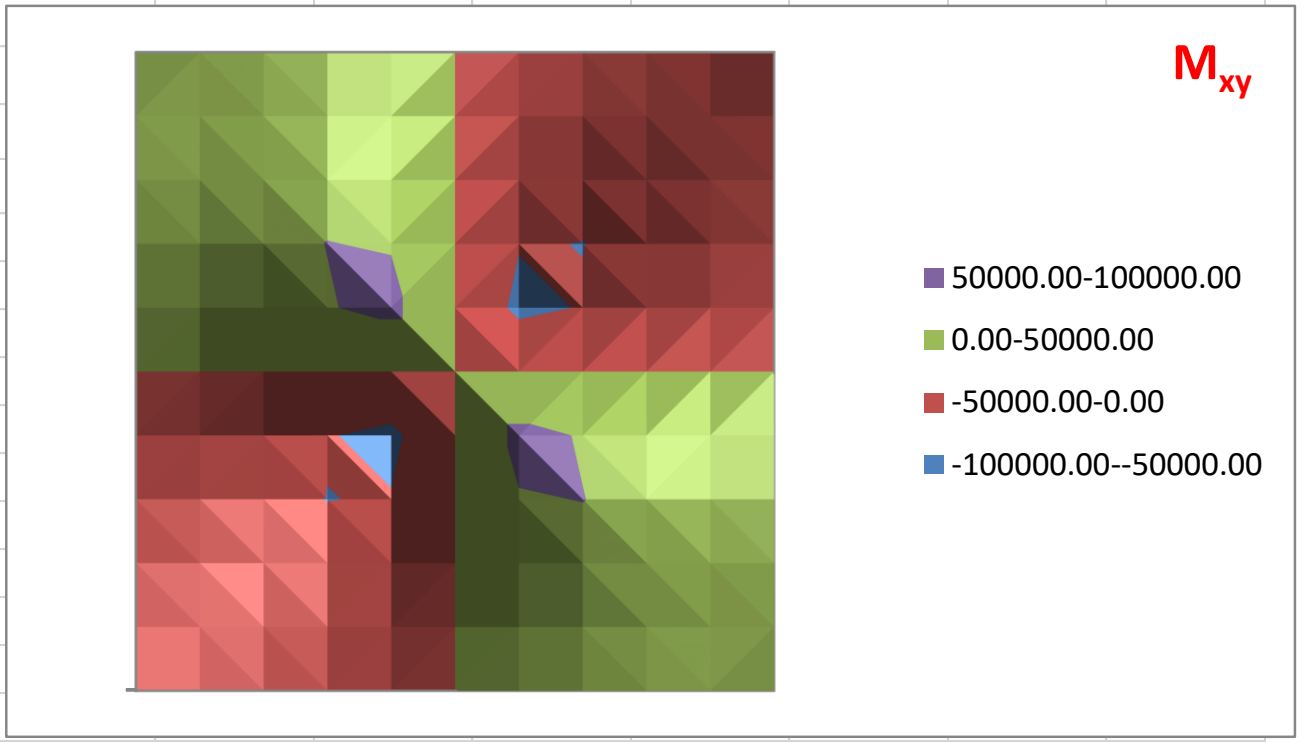
E=	31476	MPa	**Self-weight multiplier**			**Center-point force**			**Thermal effects**	
s=	200	mm	γ_g=	0.00		F=	2500.00	kN	Δt=	0
ν=	0.2		g=	5.00	kN/m^2				α_c=	1.20E-05
γ=	25.0	kN/m^3							χ^*=	0.00E+00
D=	2.186E+10	Nmm								
k_0=	0.01	N/mm^3								
L_x=	5000	mm								
L_y=	5000	mm								
n=	10									
Δ=	500	mm								



Examples and Comparison

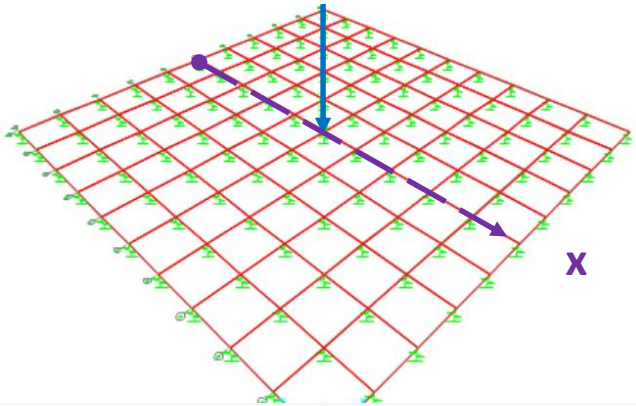
Load Case 2: concentrated force in the center point

E=	31476	MPa	**Self-weight multiplier**			**Center-point force**			**Thermal effects**		
s=	200	mm	γ_g=	0.00		F=	2500.00	kN	Δt=	0	
ν=	0.2		g=	5.00	kN/m^2				α_c=	1.20E-05	
γ=	25.0	kN/m^3							χ^*=	0.00E+00	
D=	2.186E+10	Nmm									
k_0=	0.01	N/mm^3									
L_x=	5000	mm									
L_y=	5000	mm									
n=	10										
Δ=	500	mm									

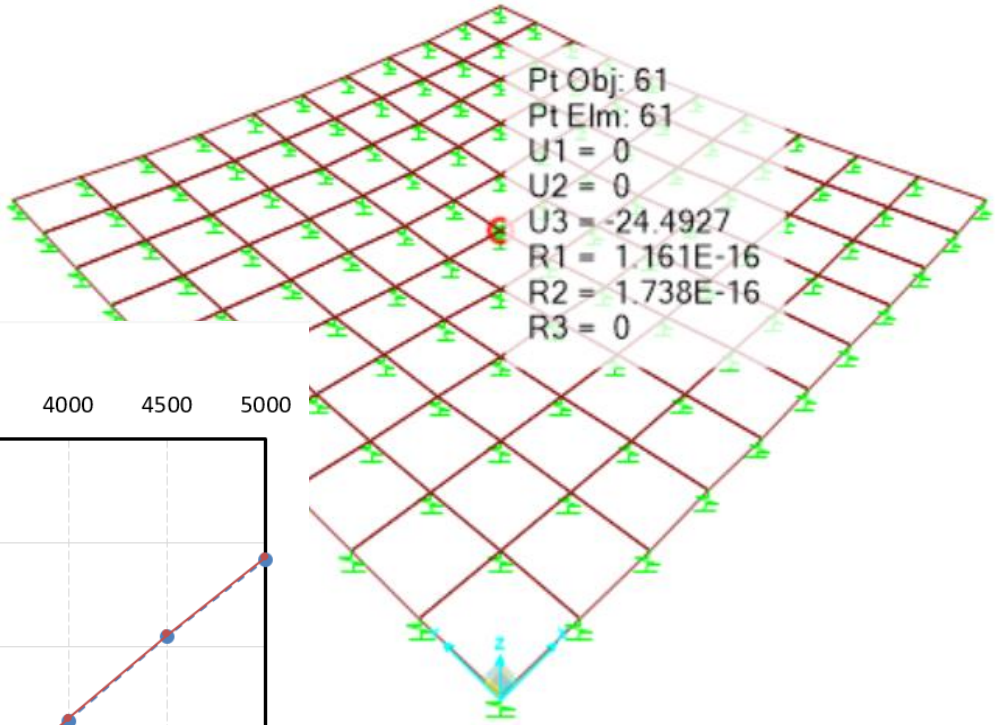


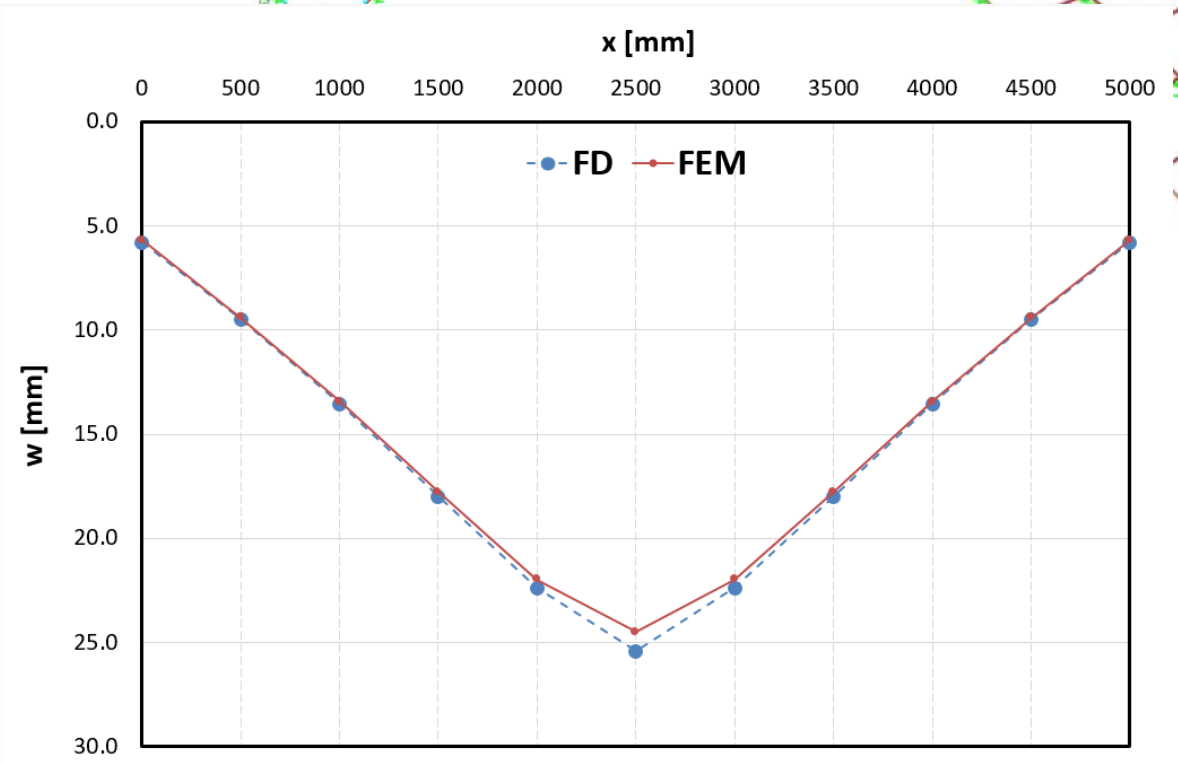
Examples and Comparison

Load Case 2: concentrated force in the center point



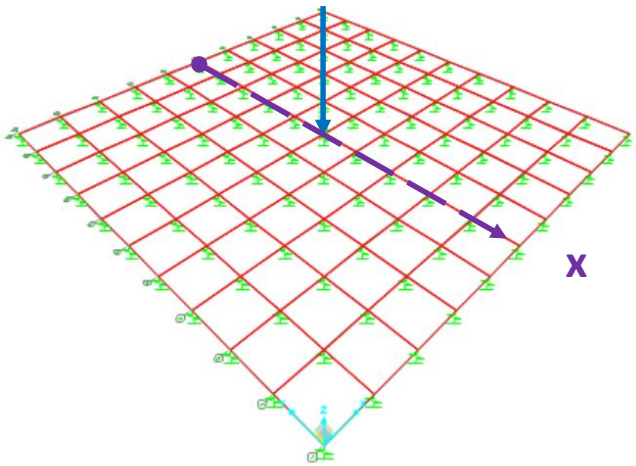
FEM Solution (SAP2000)



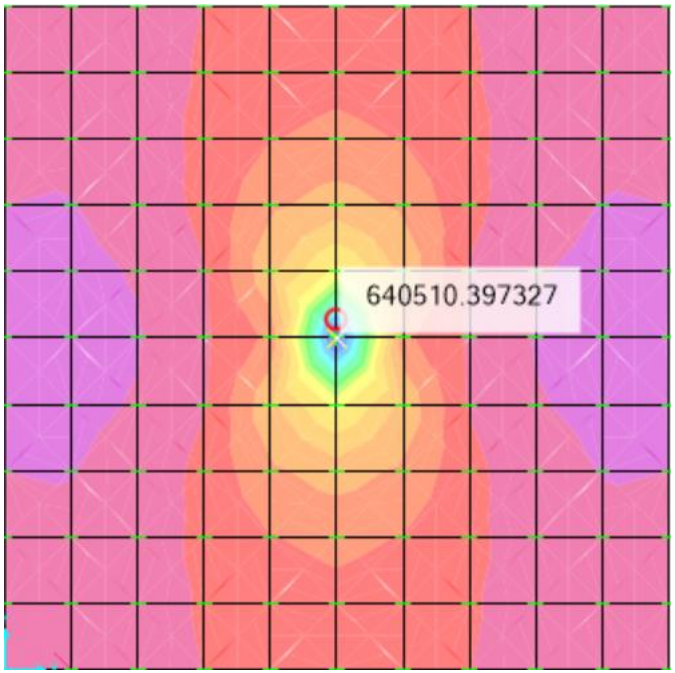


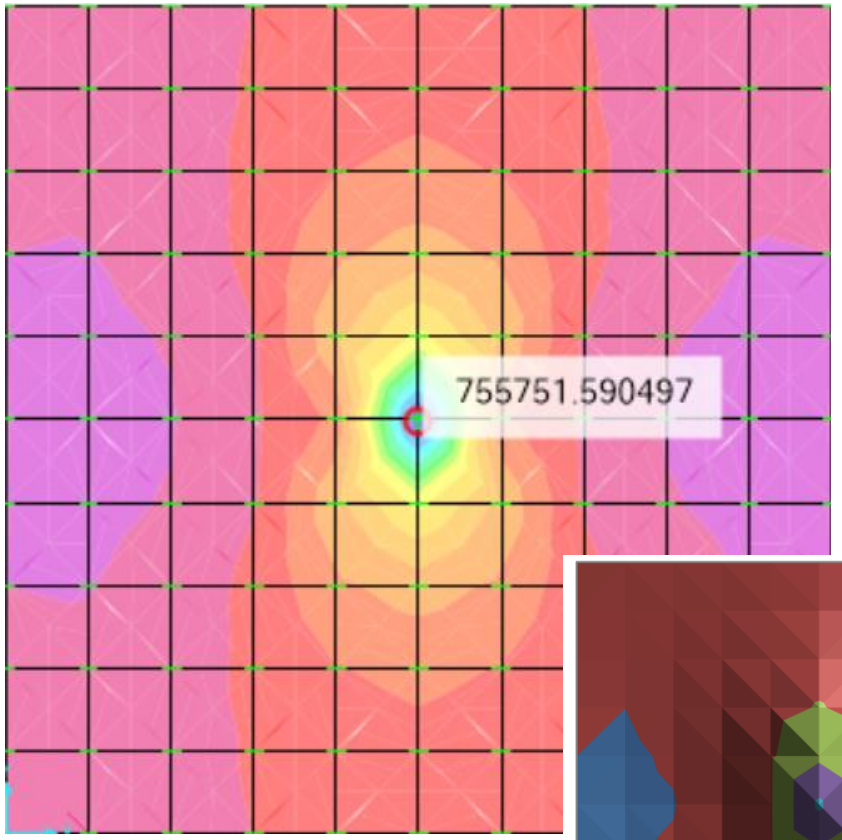
Examples and Comparison

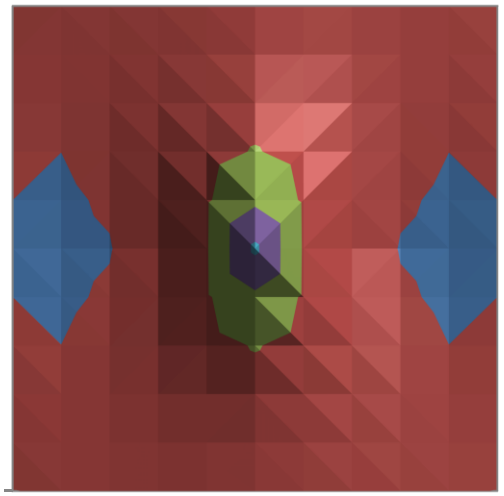
Load Case 2: concentrated force in the center point



FEM Solution (SAP2000)



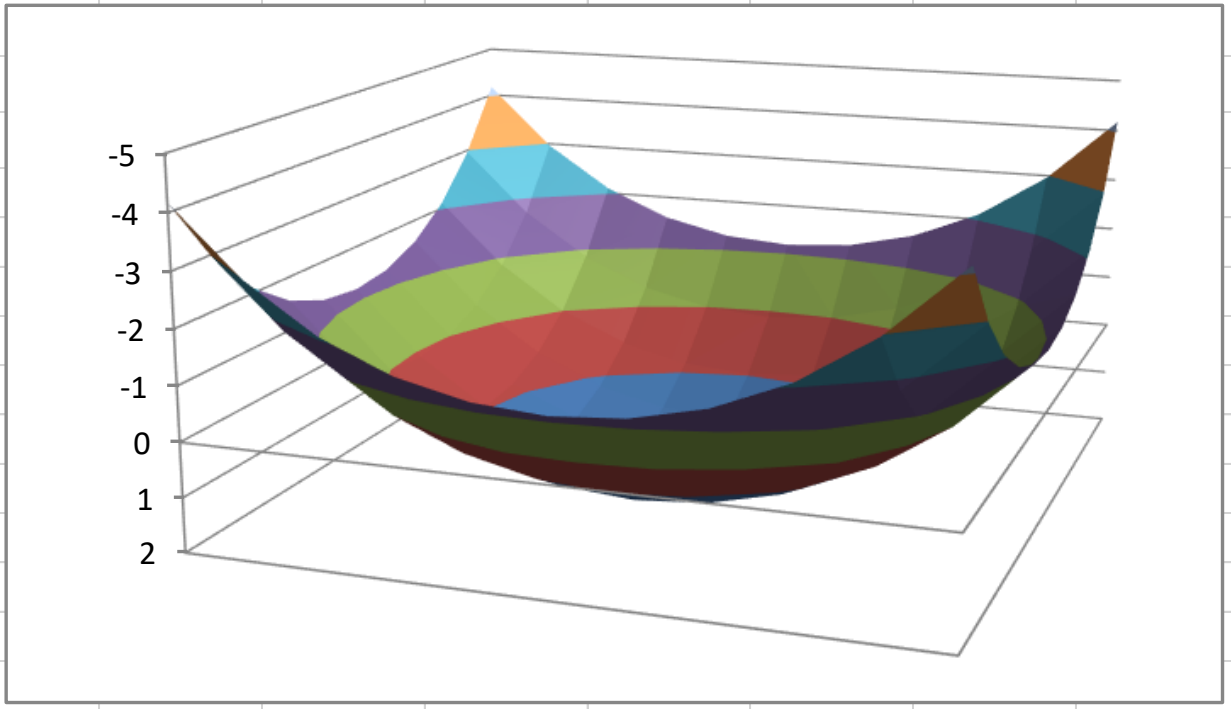




Examples and Comparison

Load Case 3: thermal effects

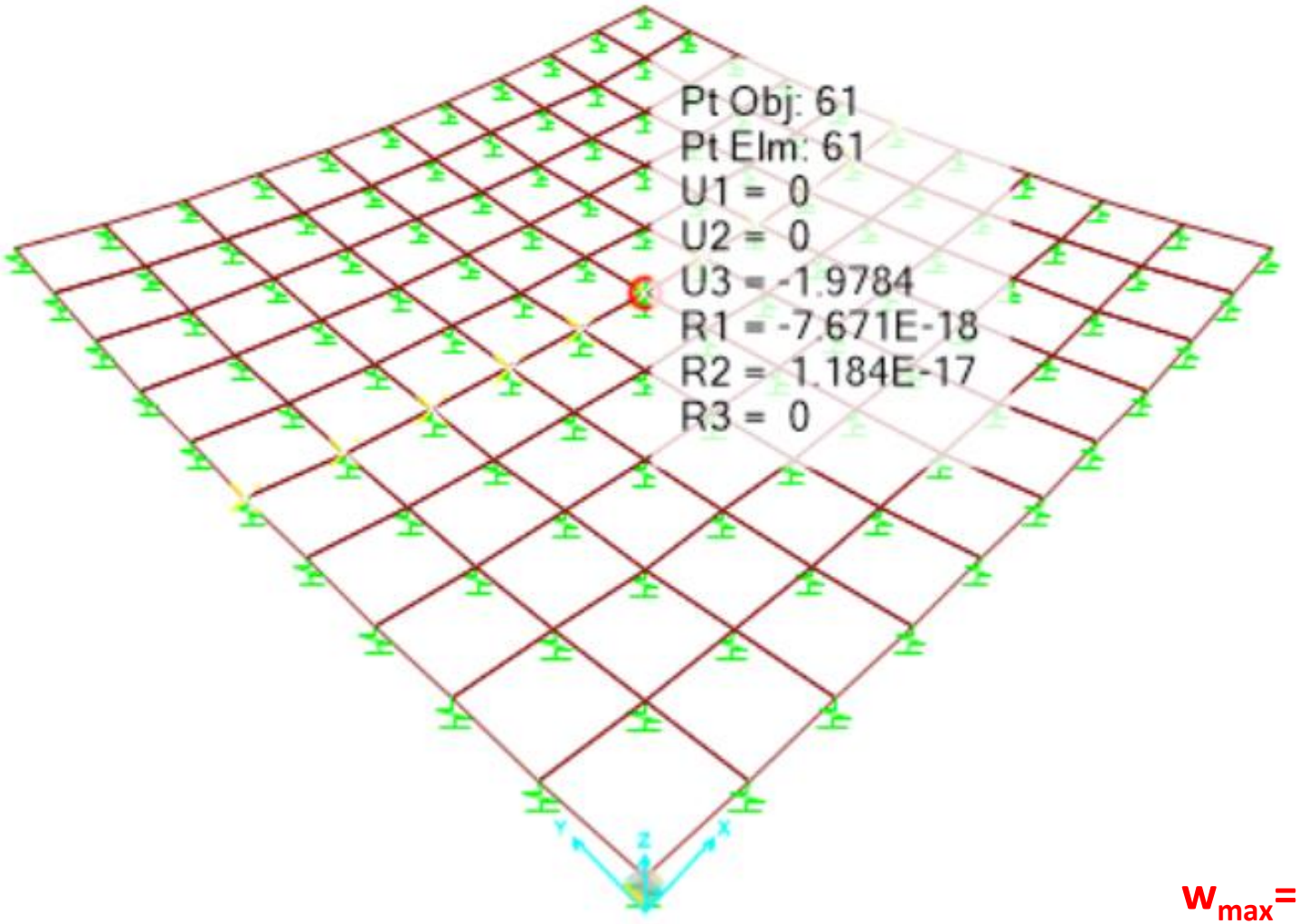
E=	31476	MPa	**Self-weight multiplier**			**Center-point force**			**Thermal effects**	
s=	200	mm	γ_g=	0.00		F=	0.00	kN	Δt=	- 25
ν=	0.2		g=	5.00	kN/m^2				α_c=	1.20E-05
γ=	25.0	kN/m^3							χ^*=	1.50E-06
D=	2.186E+10	Nmm								
k_0=	0.01	N/mm^3								
L_x=	5000	mm								
L_y=	5000	mm								
n=	10									
Δ=	500	mm								



w_{max}=1.959 mm

Examples and Comparison

Load Case 3: thermal effects

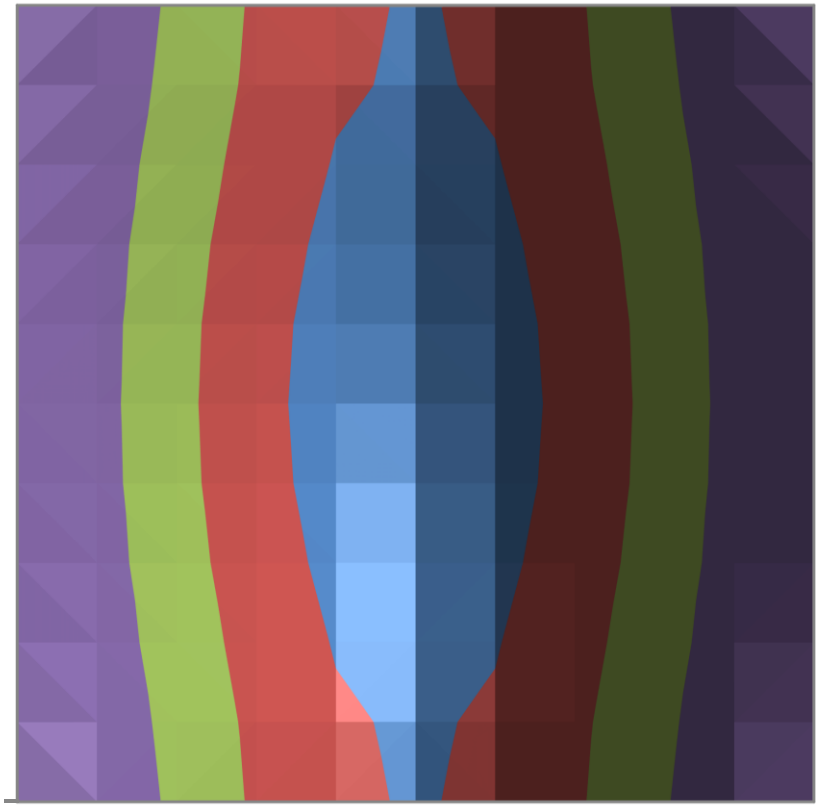


w_{max}=1.959 mm

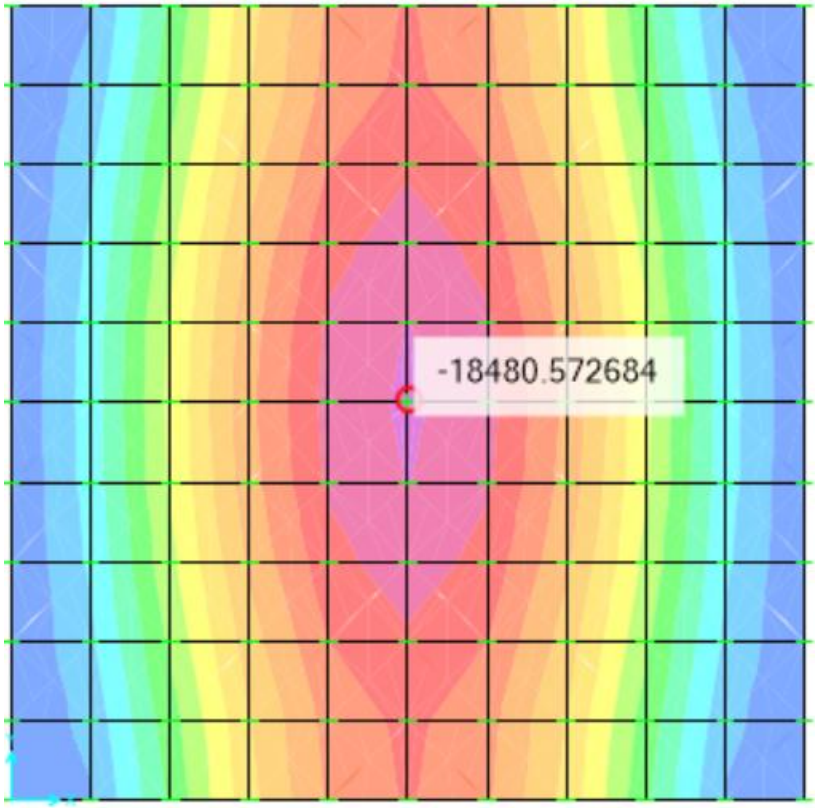
Examples and Comparison

Load Case 3: thermal effects

Finite Difference Method



Finite Element Method

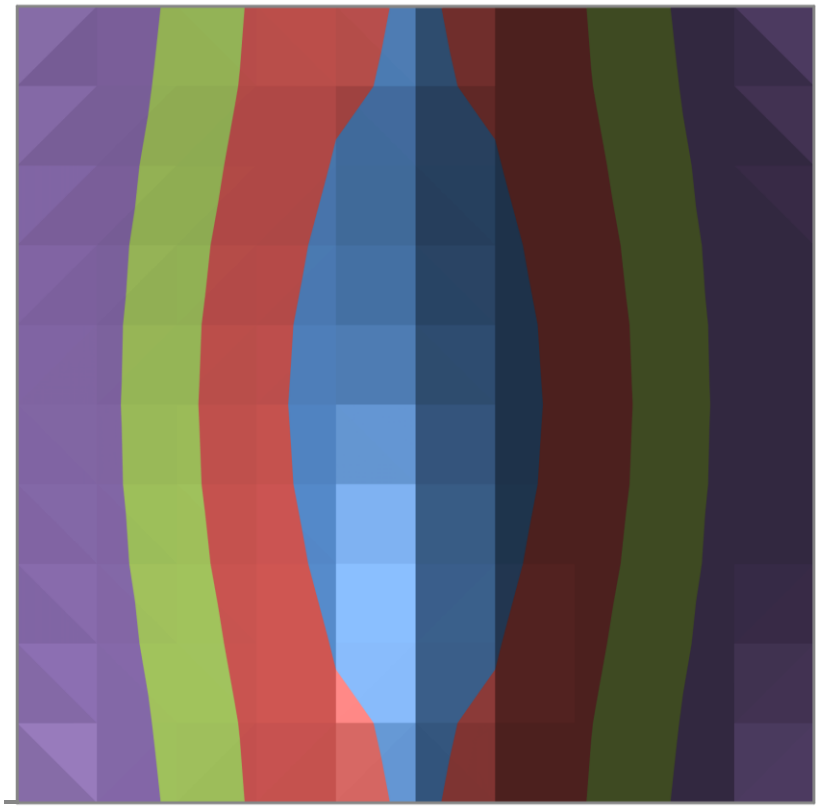


$M_{x,max}$=18306 Nmm/mm

Examples and Comparison

Load Case 3: thermal effects

Finite Difference Method



$M_{x,max}$=18306 Nmm/mm

$\sigma_{x,max}=6M_{x,max}/s^2$=2.75 MPa

The End

Thank you for your attention

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